

MTH 204

Quiz 6

20 Oct 2006

Name: Key

Sec B.

Follow the directions carefully.
Please write neatly and show
all your work in order to get
full credit. There is a table of
Laplace transforms on the
back page. If you get stuck,
feel free to ask me for help.

LEAD - Thursdays 5-7PM
CSF G5D

Exam 3: 27 Oct
4.6-

Use the Laplace transform to solve the initial value problem $y'' + 9y = e^t$ $\begin{cases} y(0) = 0 \\ y'(0) = 0 \end{cases}$

$$\mathcal{L}\{y''\} + 9\mathcal{L}\{y\} = \mathcal{L}\{e^t\}$$

$$s^2 Y(s) - sy(0) - y'(0) + 9Y(s) = \frac{1}{s-1}$$

$$(s^2 + 9)Y(s) = \frac{1}{s-1}$$

$$Y(s) = \frac{1}{(s^2 + 9)(s-1)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+9} = \frac{\frac{1}{10}}{s-1} - \frac{\frac{1}{10}s + \frac{1}{10}}{s^2+9}$$

$$1 = A(s^2+9) + Bs(s-1) + C(s-1)$$

$$s=1 \Rightarrow 1 = 10A \Rightarrow A = \frac{1}{10}$$

$$s=0 \Rightarrow 1 = \frac{1}{10}(9) + 0 - C \Rightarrow C = -\frac{1}{10}$$

$$s=-1 \Rightarrow 1 = \frac{1}{10}(10) + 2B - \frac{1}{10}(-2) \Rightarrow B = -\frac{1}{10}$$

$$\text{Then } y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$= \frac{1}{10} \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{1}{10} \mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - \frac{1}{10} \mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\left(\frac{3}{3}\right)\right\}$$

$$= \frac{1}{10} e^t - \frac{1}{10} \cos(3t) - \frac{1}{30} \sin(3t)$$