

MTH 204
Quiz 6
27 Feb 2009

Name: Key

Section: C or F (circle one)

Read the directions carefully.

Write neatly in pencil and show all your work
(you will only get credit for what you put on paper).

You may use your homework solutions.

If you get stuck, free feel to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Consider the DE $x^3 y'' + x y' - 4y = 4x^{-2}$, $x > 0$.

a. Classify the DE.

2nd order, linear, VC, NH (Cauchy-Euler)

b. What method(s) can you use to solve this DE?

{ MUC
VOP

c. Note that $y_h(x) = c_1 x^2 + c_2 x^{-2}$. Find the general sol.

1. $y_h(x) = c_1 x^2 + c_2 x^{-2}$

2. $W(y_1, y_2) = \begin{vmatrix} x^2 & x^{-2} \\ 2x & -2x^{-3} \end{vmatrix} = -2x^{-1} - 2x^{-1} = -4x^{-1}$

3. Std form: $y'' + \frac{1}{x} y' - \frac{4}{x^2} y = 4x^{-4} = f(x)$

4. Assume $y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$

"5" Plug y_p in

6. Cramer's Rule

$$W_1 = \begin{vmatrix} 0 & x^{-2} \\ 4x^{-4} & - \end{vmatrix} = -4x^{-6}$$

$$W_2 = \begin{vmatrix} x^2 & 0 \\ - & 4x^{-4} \end{vmatrix} = 4x^{-2}$$

$$u_1'(x) = \frac{W_1}{W(y_1, y_2)} = \frac{-4x^{-6}}{-4x^{-1}} = x^{-5}$$

$$u_2'(x) = \frac{W_2}{W(y_1, y_2)} = \frac{4x^{-2}}{-4x^{-1}} = -x^{-1}$$

7. Integrate & plug in

$$u_1(x) = \int x^{-5} dx = -\frac{1}{4} x^{-4} + \cancel{C_1}$$

$$u_2(x) = -\int x^{-1} dx = -\ln x + K_2$$

$$\Rightarrow y_p(x) = -\frac{1}{4} x^{-4} (x^2) - \ln x (x^{-2})$$

$$= -\frac{1}{4} x^{-2} - x^{-2} \ln x$$

↳ absorbed by y_2

8. Check if any terms of y_p are absorbed

$$y_p(x) = -x^2 \ln x$$

9. GS: $y(x) = y_h(x) + y_p(x)$

$$= c_1 x^2 + c_2 x^{-2} - x^{-2} \ln x$$

Bonus (3pts): A 64 lb object stretches a spring $\frac{4}{3}$ ft. Initially, the object starts 1 ft below equilibrium position with an upward velocity of 2 ft/s. The subsequent motion offers a damping force numerically 3 times the instantaneous velocity. Set up, but don't solve the IVP with an external force of $g(t) = 5 \sin(t)$.
 $m = \frac{64}{32} = 2$ slugs

$$\beta = 3$$

$$64 = \frac{4}{3} K$$

$$\Rightarrow K = 48 \text{ lb/ft}$$

$$2y'' + 3y' + 48y = 5 \sin(t)$$

$$\begin{cases} y(0) = 1 \\ y'(0) = -2 \end{cases}$$