

MTH 204

Quiz 7

11 Nov 2005

Name: Key

Section: A/C

Follow the directions carefully.
Show all your work to get full
credit. Please write neatly in pencil.
There is a Laplace transform table on
the back page.

LEAD session - Mondays
6 PM Rolla G4.

Test 4 - 21 Nov

Use the Laplace transform to solve the following IVP.
 $y'' + 3y' + 2y = f(t)$, subject to $\begin{cases} y(0) = 0 \\ y'(0) = 0 \end{cases}$

where $f(t) = \begin{cases} 1, & 0 \leq t < 10 \\ 0, & t \geq 10 \end{cases}$

$$f(t) = 1(u(t-0) - u(t-10)) - 0u(t-10) = 1 - u(t-10).$$

$$y'' + 3y' + 2y = 1 - u(t-10)$$

$$\mathcal{L}\{y'' + 3y' + 2y\} = \mathcal{L}\{1 - u(t-10)\}$$

$$s^2 Y(s) - sy(0) - y'(0) + 3(sY(s) - y(0)) + 2Y(s) = \frac{1}{s} - \frac{e^{-10s}}{s}$$

$$(s^2 + 3s + 2)Y(s) = \frac{1}{s} - \frac{e^{-10s}}{s}$$

$$Y(s) = \underbrace{\frac{1}{s(s+1)(s+2)}}_{F(s)} - \underbrace{\frac{e^{-10s}}{s(s+1)(s+2)}}_{e^{-10s}F(s)}$$

$$F(s) = \frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} = \frac{\frac{1}{2}}{s} - \frac{1}{s+1} + \frac{\frac{1}{2}}{s+2}$$

$$1 = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

$$s=0 \quad 1 = 2A \Rightarrow A = \frac{1}{2}$$

$$s=-1 \quad 1 = -B \Rightarrow B = -1$$

$$s=-2 \quad 1 = 2C \Rightarrow C = \frac{1}{2}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1} + \frac{1}{s+2}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$$

$$= \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)(s+2)} - \frac{e^{-10s}}{s(s+1)(s+2)}\right\}$$

$$= \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t} - \left(\frac{1}{2} - e^{-(t-10)} + \frac{1}{2}e^{-2(t-10)}\right)u(t-10)$$