MTH 204 Name Key

Quiz 7 Sec B

3 Nov 2006

Follow the directions carefully. Please write neatly in pencil. Show all your work in order to get full credit. There is a copy of the Laplace transform table on the back page. If you get stuck, feel free to ask me for help.

LEAD - Thursdays 5-7 PM
CSF G5D

1. Find $\mathcal{L}\{te^{2t}\sin(6t)\}$

$$\mathcal{L}\{te^{2t}\sin(6t)\} = -\frac{d}{ds}\mathcal{L}\{e^{2t}\sin(6t)\}$$

$$= -\frac{d}{ds}\mathcal{L}\{\sin(6t)\}\Big|_{s\to s-2}$$

$$= -\frac{d}{ds}\left[\frac{6}{s^2+6^2}\Big|_{s\to s-2}\right]$$

$$= -\frac{d}{ds}\left[\frac{6}{(s-2)^2+36}\right]$$

$$= (-1)\left[\frac{((s-2)^2+36)(0) - 6(2(s-2))}{((s-2)^2+36)^2}\right]$$

$$= \frac{12s-24}{((s-2)^2+36)^2}$$

2 Use the Laplace transform to solve $f(t) = t - \int_0^t f(\tau)d\tau$ for $f(t)$. Use the convolution technique to find the inverse.

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t\} - \mathcal{L}\{\int_0^t f(\tau)d\tau\}$

$F(s) = \frac{1}{s^2} - \mathcal{L}\{f(t) * 1\} = \frac{1}{s^2} - \mathcal{L}\{f(t)\}\mathcal{L}\{1\} = \frac{1}{s^2} - \frac{F(s)}{s}$

$\Rightarrow F(s) + \frac{1}{s}F(s) = \left(\frac{s+1}{s}\right)F(s) = \frac{1}{s^2}$

$F(s) = \left(\frac{s}{s+1}\right)\left(\frac{1}{s^2}\right) = \frac{1}{s(s+1)}$

$G(s) = \frac{1}{s} \qquad H(s) = \frac{1}{s+1}$

Then $f(t) = \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{G(s)\} * \mathcal{L}^{-1}\{H(s)\}$

$= 1 * e^{-t}$

$= \int_0^t e^{-(t-\tau)}\,d\tau$

$= e^{-t}\int_0^t e^{\tau}d\tau$

$= e^{-t}\,e^{\tau}\Big|_0^t$

$= e^{-t}[e^t - 1]$

$= 1 - e^{-t}$