

MTH 204

Quiz 7

7 April 2006

Name: Key

Section: D/E

Follow the direction carefully
Please neatly in pencil and show
ALL your work in order to get
full credit. A table of Laplace
transforms is on the back page.
If you have any trouble, feel free
to ask me for help.

LEAD Session - Wednesdays

6:00-7:30 PM

CSF G5D

Test 4: 28 April

7.4-

1. Find $\mathcal{L}\{te^{-3t}\cos(3t)\}$

$$\mathcal{L}\{te^{-3t}\cos(3t)\} = -\frac{d}{ds} \mathcal{L}\{e^{-3t}\cos(3t)\}$$

ds

$$= -\frac{d}{ds} \mathcal{L}\{\cos(3t)\} \Big|_{s \rightarrow s-(-3)}$$

ds

$$= -\frac{d}{ds} \left[\frac{s}{s^2+9} \right] \Big|_{s \rightarrow s+3}$$

$$= -\frac{d}{ds} \left[\frac{s+3}{(s+3)^2+9} \right]$$

$$= -\frac{[(s+3)^2+9](1) - (s+3)(2(s+3))}{((s+3)^2+9)^2}$$

$$= \frac{(s+3)^2 - 9}{((s+3)^2+9)^2}$$

$$= \frac{s^2+6s}{((s+3)^2+9)^2}$$

2 Find $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s-1)}\right\}$ using convolutions.

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2(s-1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} * \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

$$= t * e^t$$

$$= \int_0^t \tau e^{t-\tau} d\tau$$

$$= e^t \int_0^t \tau e^{-\tau} d\tau$$

$$u = \tau \quad dv = e^{-\tau}$$

$$du = d\tau \quad v = -e^{-\tau}$$

$$= e^t \left(-\tau e^{-\tau} \Big|_0^t - \int_0^t -e^{-\tau} d\tau \right)$$

$$= e^t \left(-te^{-t} + 0 + \int_0^t e^{-\tau} d\tau \right)$$

$$= e^t \left(-te^{-t} - e^{-\tau} \Big|_0^t \right)$$

$$= e^t \left(-te^{-t} - e^{-t} + 1 \right)$$

$$= -t - 1 + e^t$$