

MTH 204
Quiz 7
6 Apr 2007

Name: Key

Section: B or C (circle one)

Read the directions carefully.

Write neatly in pencil and show all your work
(you will only get credit for what you put on paper).

You may use your homework solutions.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Exam 3: 20 April
Sections 7.4 -

By hand, find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$.

$$\begin{aligned} 0 &= \det(A - \lambda I) = \lambda^2 - (1-2)\lambda + (-2) - 4 \\ &= \lambda^2 + \lambda - 6 \\ &= (\lambda + 3)(\lambda - 2) \\ \Rightarrow \lambda_1 &= -3 \\ \lambda_2 &= 2 \end{aligned}$$

For $\lambda_1 = -3$

$$(A - \lambda_1 I) \vec{K}_1 = \vec{0}, \vec{K}_1 \neq \vec{0}$$

$$\begin{bmatrix} 1 - (-3) & 1 \\ 4 & -2 - (-3) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & | & 0 \\ 4 & 1 & | & 0 \end{bmatrix} \quad R_2 = R_1$$

$$4u_1 + u_2 = 0$$

$$u_2 = -4u_1 \xrightarrow{FV=1}$$

$$\vec{K}_1 = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ -4u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$\text{eigenpair } \left\{ -3, \begin{bmatrix} 1 \\ -4 \end{bmatrix} \right\}$$

For $\lambda_2 = 2$

$$(A - \lambda_2 I) \vec{K}_2 = \vec{0}, \vec{K}_2 \neq \vec{0}$$

$$\begin{bmatrix} 1-2 & 1 \\ 4 & -2-2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & | & 0 \\ 4 & -4 & | & 0 \end{bmatrix} \quad R_2 = -4R_1$$

$$-v_1 + v_2 = 0$$

$$v_2 = v_1 \xrightarrow{FV=1}$$

$$\vec{K}_2 = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{eigenpair } \left\{ 2, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$