

MTH 204  
Quiz 7  
6 Mar 2009

Name: Key

Section: C or F (circle one)

Read the directions carefully.

Write neatly in pencil and **show all your work**  
**(you will only get credit for what you put on paper).**

**You may use your homework solutions.**

**If you get stuck, free feel to ask me for help.**

1. Solve the following Cauchy-Euler equations when  $x > 0$ .

a.  $x^2 y'' + 7xy' + 9y = 0$

Assume  $y(x) = x^m$

$$y'(x) = mx^{m-1}$$

$$y''(x) = m(m-1)x^{m-2}$$

$$x^2 m(m-1)x^{m-2} + 7xm^{m-1} + 9x^m = 0$$

$$x^m [m(m-1) + 7m + 9] = 0$$

$$m^2 + (7-1)m + 9 = 0$$

$$(m+3)^2 = 0$$

$$\Rightarrow m = -3, -3$$

$$y(x) = C_1 x^{-3} + C_2 x^{-3} \ln x$$

b.  $x^2 y'' + 5xy' + 5y = 0$

Assume  $y(x) = x^m$

$$y'(x) = mx^{m-1}$$

$$y''(x) = m(m-1)x^{m-2}$$

$$\Rightarrow x^m [m(m-1) + 5m + 5] = 0$$

$$m^2 + (5-1)m + 5 = 0$$

$$m = \frac{-4 \pm \sqrt{16 - 4(1)(5)}}{2(1)} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

$$y(x) = x^{-2} [C_1 \cos(\ln x) + C_2 \sin(\ln x)]$$

2. Use the definition  $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$  to find the Laplace transform of  $f(t) = te^{4t}$ . For what values of  $s$  does  $\mathcal{L}\{f(t)\} = F(s)$ ?

$$\mathcal{L}\{te^{4t}\} = \int_0^{\infty} e^{-st} te^{4t} dt$$

$$= \int_0^{\infty} e^{-(s-4)t} t dt$$

$$u = t$$

$$du = dt$$

$$dv = e^{-(s-4)t} dt$$

$$v = \frac{-1}{s-4} e^{-(s-4)t}$$

$$= \frac{-t}{s-4} e^{-(s-4)t} \Big|_0^{\infty} + \left( \frac{1}{s-4} \right) \int_0^{\infty} e^{-(s-4)t} dt$$

$$= \underbrace{0 - 0}_{s-4 > 0} - \frac{1}{(s-4)^2} e^{-(s-4)t} \Big|_0^{\infty}$$

$$= 0 - - \frac{1}{(s-4)^2}$$

$$= \frac{1}{(s-4)^2}, \text{ provided } s-4 > 0$$

$s > 4$

Bonus (1pt): When and where is LEAD?

Thursdays, 5-7 PM

CSF G5D