

MTH 204

Quiz 8

18 Nov 2005

Name: Key

Section: A1C

Follow the directions carefully.
Show all your work to get full
credit. Please write neatly in pencil.
There is a Laplace transform table on
the back page.

LEAD session - Mondays

6 PM Rolla G4.

Test 4 - 21 Nov

Ch 7 & 8.1.

1. Find $\mathcal{L}\{te^{2t}\sin(6t)\}$

$$\begin{aligned}
 \mathcal{L}\{te^{2t}\sin(6t)\} &= -\frac{d}{ds} \mathcal{L}\{e^{2t}\sin(6t)\} \\
 &= -\frac{d}{ds} \mathcal{L}\{\sin(6t)\}_{s \rightarrow s-2} \\
 &= -\frac{d}{ds} \left[\frac{6}{s^2+36} \right]_{s \rightarrow s-2} \\
 &= -\frac{d}{ds} \left[\frac{6}{(s-2)^2+36} \right] \\
 &= - \left[\frac{((s-2)^2+36)(6)' - 6((s-2)^2+36)'}{((s-2)^2+36)^2} \right] \\
 &= - \left[\frac{0 - 6(2(s-2))}{((s-2)^2+36)^2} \right] \\
 &= \frac{12s-24}{((s-2)^2+36)^2}
 \end{aligned}$$

2. Solve the IVP $y'' + y = \delta(t-2\pi) + \delta(t-4\pi)$, $\begin{cases} y(0) = 1 \\ y'(0) = 0 \end{cases}$

$$\begin{aligned}
 \mathcal{L}\{y'' + y\} &= \mathcal{L}\{\delta(t-2\pi) + \delta(t-4\pi)\} \\
 \mathcal{L}\{y''\} + \mathcal{L}\{y\} &= \mathcal{L}\{\delta(t-2\pi)\} + \mathcal{L}\{\delta(t-4\pi)\} \\
 s^2 Y(s) - sy(0) - y'(0) + Y(s) &= e^{-2\pi s} + e^{-4\pi s}
 \end{aligned}$$

$$(s^2+1)Y(s) = e^{-2\pi s} + e^{-4\pi s} + s$$

$$Y(s) = \frac{e^{-2\pi s}}{s^2+1} + \frac{e^{-4\pi s}}{s^2+1} + \frac{s}{s^2+1}$$

$$F(s) = \frac{1}{s^2+1} \Rightarrow f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin(t).$$

$$\begin{aligned}
 y(t) &= \mathcal{L}^{-1}\{Y(s)\} \\
 &= \mathcal{L}^{-1}\left\{\frac{e^{-2\pi s}}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{e^{-4\pi s}}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} \\
 &= \sin(t-2\pi)U(t-2\pi) + \sin(t-4\pi)U(t-4\pi) + \cos(t) \\
 &= \sin(t)U(t-2\pi) + \sin(t)U(t-4\pi) + \cos(t)
 \end{aligned}$$