

MTH 204

Quiz 8

10 Nov 2006

Name Key
Sec B.

Follow the directions carefully.
Please write neatly in pencil.
You must show all your work
to get full credit. If you get
stuck, feel free to ask me
for help.

LEAD - Thursdays 5-7 PM
CSF G5D.

Test 3 - 17 Nov
7.4 - 8.2

By hand, calculate the eigenvalues and eigenvectors of the following matrix.

$$A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$$

$$0 = \det \begin{vmatrix} 5-\lambda & -1 \\ 3 & 1-\lambda \end{vmatrix} = (5-\lambda)(1-\lambda) + 3 = \lambda^2 - 6\lambda + 8 = (\lambda-2)(\lambda-4)$$

$$\Rightarrow \lambda_1 = 2, \lambda_2 = 4.$$

$$\text{For } \lambda_1 = 2 \Rightarrow (A - \lambda_1 I) \vec{K}_1 = \vec{0}$$

$$\begin{bmatrix} 5-2 & -1 & | & 0 \\ 3 & 1-2 & | & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 & | & 0 \\ 3 & -1 & | & 0 \end{bmatrix} \Rightarrow 3k_1 - k_2 = 0$$

$$3k_1 = k_2 \\ \hookrightarrow FV = 1$$

$$\Rightarrow \vec{K}_1 = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} k_1 \\ 3k_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \left\{ 2, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$$

$$\text{For } \lambda_2 = 4 \Rightarrow (A - \lambda_2 I) \vec{K}_2 = \vec{0}$$

$$\begin{bmatrix} 5-4 & -1 & | & 0 \\ 3 & 1-4 & | & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & | & 0 \\ 3 & -3 & | & 0 \end{bmatrix} \Rightarrow k_1 - k_2 = 0$$

$$k_1 = k_2$$

$$\hookrightarrow FV = 1$$

$$\Rightarrow \vec{K}_2 = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} k_2 \\ k_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \left\{ 4, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

Bonus (2pts): What is the general solution for $\vec{x}' = A\vec{x}$?

$$\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t}$$