

MTH204

Quiz 8

14 April 2006

Name: Key

Section: D/E

Follow the direction carefully
Please neatly in pencil and show
All your work in order to get
full credit. A table of Laplace
transforms is on the back page.
If you have any trouble, feel free
to ask me for help.

LEAD Session - Wednesdays

6:00-7:30 PM

CSFG5D

Test 4: 28 April

7.4-

Use the Laplace transform

$$2\frac{dx}{dt} + \frac{dy}{dt} - 2x = 1$$

$$\begin{cases} x(0) = 0 \\ y(0) = 0. \end{cases}$$

$$\frac{dx}{dt} + \frac{dy}{dt} - 3x - 3y = 2$$

$$\mathcal{L}\{2x' + y' - 2x\} = \mathcal{L}\{1\}$$

$$\mathcal{L}\{x' + y' - 3x - 3y\} = \mathcal{L}\{2\}$$

$$\begin{cases} 2sX(s) - 2x(0) + sY(s) - y(0) - 2X(s) = \frac{1}{s} \end{cases}$$

$$\begin{cases} sX(s) - x(0) + sY(s) - y(0) - 3X(s) - 3Y(s) = \frac{2}{s} \end{cases}$$

$$(s-3) \begin{cases} 2(s-1)X(s) + sY(s) = \frac{1}{s} \\ -s((s-3)X(s) + (s-3)Y(s)) = \frac{2}{s} \end{cases}$$

$$2(s-1)(s-3)X(s) - s(s-3)X(s) = \frac{s-3}{s} - \frac{2s}{s}$$

$$(s-2)(s-3)X(s) = \frac{-s-3}{s}$$

$$\Rightarrow X(s) = \frac{-s-3}{s(s-2)(s-3)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s-3}$$

$$-s-3 = A(s-2)(s-3) + Bs(s-3) + Cs(s-2)$$

$$s=0 \Rightarrow -3 = 6A \Rightarrow A = -\frac{1}{2}$$

$$s=2 \Rightarrow -5 = -2B \Rightarrow B = \frac{5}{2}$$

$$s=3 \Rightarrow -6 = 3C \Rightarrow C = -2$$

$$= \frac{-\frac{1}{2}}{s} + \frac{\frac{5}{2}}{s-2} - \frac{2}{s-3}$$

$$x(t) = \frac{-1}{2} \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + \frac{5}{2} \mathcal{L}^{-1} \left\{ \frac{1}{s-2} \right\} - 2 \mathcal{L}^{-1} \left\{ \frac{1}{s-3} \right\}$$

$$= \frac{-1}{2} + \frac{5}{2} e^{2t} - 2e^{3t}$$

$$(s-3) \begin{cases} 2(s-1)X(s) + sY(s) = \frac{1}{s} \\ (s-3)X(s) + (s-3)Y(s) = \frac{2}{s} \end{cases}$$

$$s(s-3)Y(s) - 2(s-1)(s-3)Y(s) = \frac{s-3}{s} - \frac{4(s-1)}{s} = \frac{-3s+1}{s}$$

$$-(s-2)(s-3)Y(s) = \frac{-3s+1}{s}$$

$$Y(s) = \frac{3s-1}{s(s-2)(s-3)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s-3} = \frac{-\frac{1}{6}}{s} - \frac{\frac{5}{2}}{s-2} + \frac{\frac{8}{3}}{s-3}$$

$$3s-1 = A(s-2)(s-3) + Bs(s-3) + Cs(s-2)$$

$$s=0 \Rightarrow -1 = 6A \Rightarrow A = -\frac{1}{6}$$

$$s=2 \Rightarrow 5 = -2B \Rightarrow B = -\frac{5}{2}$$

$$s=3 \Rightarrow 8 = 3C \Rightarrow C = \frac{8}{3}$$

$$\text{Then } y(t) = \frac{-1}{6} \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} - \frac{5}{2} \mathcal{L}^{-1} \left\{ \frac{1}{s-2} \right\} + \frac{8}{3} \mathcal{L}^{-1} \left\{ \frac{1}{s-3} \right\}$$

$$= \frac{-1}{6} - \frac{5}{2} e^{2t} + \frac{8}{3} e^{3t}$$

$$\text{Now } x(t) = \frac{-1}{2} + \frac{5}{2} e^{2t} - 2e^{3t}$$

$$y(t) = \frac{-1}{6} - \frac{5}{2} e^{2t} + \frac{8}{3} e^{3t}$$