MTH 204
Quiz 8
13 Apr 2007

Name: Key

Section: B or C (circle one)

Read the directions carefully.
Write neatly in pencil and **show all your work**
(you will only get credit for what you put on paper).
You may use your homework solutions.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Exam 3: 20 April
Sections 7.4 -

Solve the initial value problem $\vec{x}' = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}\vec{x}, \quad \vec{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

$$0 = \det(A-\lambda I) = \lambda^2 - \operatorname{Tr}A\,\lambda + \det A$$
$$= \lambda^2 - 6\lambda + 5 - (-3)$$
$$= \lambda^2 - 6\lambda + 8$$
$$= (\lambda-2)(\lambda-4)$$
$$\Rightarrow \lambda_1 = 2, \quad \lambda_2 = 4.$$

For $\lambda_1 = 2$,

$(A-\lambda_1 I)\vec{K}_1 = \vec{0}, \ \vec{K}_1 \neq \vec{0}$

$$\begin{bmatrix} 5-2 & -1 \\ 3 & 1-2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 3 & -1 & 0 \\ 3 & -1 & 0 \end{array}\right] \quad R_2 = R_1$$

$3u_1 - u_2 = 0$

$u_2 = 3u_1$ ↳ FV = 1

$$\vec{K}_1 = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ 3u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \vec{x}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2t}$$

For $\lambda_2 = 4$

$(A-\lambda_2 I)\vec{K}_2 = \vec{0}, \ \vec{K}_2 \neq \vec{0}$

$$\begin{bmatrix} 5-4 & -1 \\ 3 & 1-4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 3 & -3 & 0 \end{array}\right] \quad R_2 = 3R_1$$

$v_1 - v_2 = 0$

$v_2 = v_1$ ↳ FV = 1

$$\vec{K}_2 = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \vec{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t}$$

$$\vec{x}(t) = c_1\begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2t} + c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t}$$

$$\vec{x}(0) = c_1\begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$c_1 + c_2 = 2$$
$$-(3c_1 + c_2 = -1)$$
$$-2c_1 = 3$$

$$c_1 = -\frac{3}{2} \qquad c_2 = \frac{7}{2}$$

$$\vec{x}(t) = -\frac{3}{2}\begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2t} + \frac{7}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t}.$$