

MTH 204
Quiz 8
3 Apr 2009

Name: Key
Section: C or F (circle one)

Read the directions carefully.

Write neatly in pencil and show all your work
(you will only get credit for what you put on paper).

You may use your homework solutions.
If you get stuck, free feel to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Solve the integrodifferential equation
 $y'(t) + 4 \int_0^t e^{-4\tau} y(t-\tau) d\tau = 4, \quad y(0) = 1$

$$\Rightarrow y'(t) + 4(e^{-4t} * y(t)) = 4$$

1. Take Laplace

$$\mathcal{L}\{y'(t)\} + 4 \underbrace{\mathcal{L}\{e^{-4t}\} \mathcal{L}\{y(t)\}} = \mathcal{L}\{4\}$$

$$\overset{1}{sY(s) - y(0)} + \frac{4Y(s)}{s+4} = \frac{4}{s}$$

2. Solve for $Y(s)$

$$\left(\frac{s(s+4)+4}{s+4} \right) Y(s) = \frac{(s+2)^2 Y(s)}{s+4} = \frac{4}{s} + 1 = \frac{s+4}{s}$$

$$\Rightarrow Y(s) = \frac{(s+4)^2}{s(s+2)^2}$$

3. Partial Fractions

$$\frac{(s+4)^2}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2} = \frac{4}{s} - \frac{3}{s+2} - \frac{2}{(s+2)^2}$$

$$\Rightarrow (s+4)^2 = A(s+2)^2 + Bs(s+2) + Cs$$

$$s=0 \Rightarrow 16 = 4A + 0 + 0 \Rightarrow A=4$$

$$s=-2 \Rightarrow 4 = 0 + 0 - 2C \Rightarrow C=-2$$

$$s=-1 \Rightarrow 9 = 4(1) - B + 2 \Rightarrow B=-3$$

4. Take inverse Laplace

$$\begin{aligned} y(t) &= \mathcal{L}^{-1}\{Y(s)\} = 4\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\} \\ &= 4 - 3e^{-2t} - 2te^{-2t} \end{aligned}$$