MTH204

Quiz 9

2 Dec 2005

Name: Key

Section: A/C

Follow the directions carefully. Show all your work neatly in pencil. You may not use a calculator to compute the eigenvalues and eigenvectors for the given system.

LEAD Session - Mondays
6PM Rolla G4

Final 15 Dec Schrenk G3
1:30 - 3:30

Consider the system $\vec{x}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \vec{x}$

a. Solve the system

b. Classify the critical point (0,0) as either a nodal sink, nodal source, or saddle point.

c. Sketch a phase plane of the system showing at least 6 trajectories.

(a) $0 = \det(A - \lambda I) = \begin{vmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{vmatrix} = (-2-\lambda)^2 - 1 = \lambda^2 + 4\lambda + 3 = (\lambda+1)(\lambda+3)$

$\begin{cases} \lambda_1 = -1 \\ \lambda_2 = -3 \end{cases} \Rightarrow$ Nodal sink (b).

For $\lambda_1 = -1$, $\left[\begin{array}{cc|c} -2-(-1) & 1 & 0 \\ 1 & -2-(-1) & 0 \end{array}\right] = \left[\begin{array}{cc|c} -1 & 1 & 0 \\ \cancel{1} & \cancel{-1} & \cancel{0} \end{array}\right]$ $\quad -k_1 + k_2 = 0$

$k_1 = k_2$

$\uparrow$ FV = 1

$\vec{K}_1 = \begin{bmatrix} k_1 \\ k_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \vec{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t}$

For $\lambda_2 = -3$, $\left[\begin{array}{cc|c} -2-(-3) & 1 & 0 \\ 1 & -2-(-3) & 0 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right]$ $\quad k_1 + k_2 = 0$

$k_1 = -k_2$

$\uparrow$ FV = 1

$\vec{K}_2 = \begin{bmatrix} -k_2 \\ k_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \Rightarrow \vec{x}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-3t}$

Then $\vec{x}(t) = c_1 \vec{x}_1 + c_2 \vec{x}_2$

$= c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-3t}$

(c.)