

MTH 204

Quiz 9

17 Nov 2006

Name Key
Sec B

Follow the directions carefully.
Please write neatly in pencil. You
must show all your work to
receive full credit. If you get
stuck, please feel free to ask
me for help.

LEAD: Thursdays 5-7PM
CSF G5D

Review Session: 28 Nov 5-7PM
CSF G5D

Test 4: 29 Nov
Sec 7.4-8.2 excluding 7.5

Find the general solution of $\vec{x}' = \begin{bmatrix} -1 & 3 \\ -3 & 5 \end{bmatrix} \vec{x}$.

$$0 = \begin{vmatrix} -1-\lambda & 3 \\ -3 & 5-\lambda \end{vmatrix} = (-1-\lambda)(5-\lambda) + 9 = \lambda^2 - 4\lambda + 4 = (\lambda - 2)^2$$

$$\Rightarrow \lambda_1 = \lambda_2 = 2$$

$$(A - \lambda I) \vec{K} = \vec{0}$$

$$\begin{bmatrix} -1-2 & 3 & | & 0 \\ -3 & 5-2 & | & 0 \end{bmatrix} = \begin{bmatrix} -3 & 3 & | & 0 \\ -3 & 3 & | & 0 \end{bmatrix} \Rightarrow -K_1 + K_2 = 0$$

$$K_1 = K_2$$

$$\hookrightarrow FV = 1$$

$$\Rightarrow \vec{K} = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \vec{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$$

$$(A - \lambda I) \vec{P} = \vec{K}$$

$$\begin{bmatrix} -1-2 & 3 & | & 1 \\ -3 & 5-2 & | & 1 \end{bmatrix} = \begin{bmatrix} -3 & 3 & | & 1 \\ -3 & 3 & | & 1 \end{bmatrix} \Rightarrow -3P_1 + 3P_2 = 1$$

$$P_2 = \frac{1}{3} + P_1 \hookrightarrow 0$$

$$\Rightarrow \vec{P} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ \frac{1}{3} + P_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{3} \end{bmatrix} + P_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{3} \end{bmatrix}$$

$$\Rightarrow \vec{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t e^{2t} + \begin{bmatrix} 0 \\ \frac{1}{3} \end{bmatrix} e^{2t}$$

$$\text{Then } \vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} t e^{2t} + \begin{bmatrix} 0 \\ \frac{1}{3} \end{bmatrix} e^{2t} \right)$$