MTH 204
Quiz 9
21 April 2006

Name: Key
Section: D/E

Follow the directions carefully. Please write neatly in pencil and show all your work in order to get full credit. You are expected to find eigenvalues and eigenvectors to the following linear system by hand. If you have any trouble, feel free to ask me for help.

LEAD Session - Wednesdays
6:00 - 7:30
CSF G5D

Test 4: 28 April
7.4 - 8.2

Final: 8 May. 1:30 - 3:30.
EE G31.

Consider the linear system $\vec{x}' = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix}\vec{x}$

a. Find the general solution.

b. Solve the IVP when $\vec{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$0 = \det(A - \lambda I) = \begin{vmatrix} -5-\lambda & 1 \\ 4 & -2-\lambda \end{vmatrix}$$
$$= (-5-\lambda)(-2-\lambda) - 4$$
$$= \lambda^2 + 7\lambda + 6$$
$$= (\lambda+1)(\lambda+6)$$

$\Rightarrow \lambda_1 = -1, \lambda_2 = -6.$

For $\lambda_1 = -1$, $(A - \lambda_1 I)\vec{K}_1 = \vec{0}$

$$\Rightarrow \left[\begin{array}{cc|c} -5-(-1) & 1 & 0 \\ 4 & -2-(-1) & 0 \end{array}\right] = \left[\begin{array}{cc|c} -4 & 1 & 0 \\ \cancel{4} & \cancel{-1} & \cancel{0} \end{array}\right] \Rightarrow -4k_1 + k_2 = 0 \Rightarrow k_2 = 4k_1$$

$$\Rightarrow \vec{K}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \quad \text{so } \vec{x}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-t} \qquad \uparrow FV = 1.$$

For $\lambda_2 = -6$, $(A - \lambda_2 I)\vec{K}_2 = \vec{0}$

$$\Rightarrow \left[\begin{array}{cc|c} -5-(-6) & 1 & 0 \\ 4 & -2-(-6) & 0 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ \cancel{4} & \cancel{4} & \cancel{0} \end{array}\right] \Rightarrow k_1 + k_2 = 0 \Rightarrow -k_1 = k_2$$

$$\Rightarrow \vec{K}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{so } \vec{x}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-6t} \qquad \uparrow FV = 1.$$

Then our general solution is $\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-6t}$

$$\vec{x}(0) = c_1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \begin{array}{l} c_1 + c_2 = 2 \\ 4c_1 - c_2 = 1 \end{array} \Rightarrow \begin{array}{l} c_1 = 3/5 \\ c_2 = 2/5 \end{array}$$

So $\vec{x}(t) = \frac{3}{5}\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-t} + \frac{2}{5}\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-6t}$