MTH 204
Quiz 9
27 Apr 2007

Name: Key

Section: B or C (circle one)

Read the directions carefully.
Write neatly in pencil and **show all your work**
(you will only get credit for what you put on paper).
You may use your homework solutions.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Final Exam: 9 May, 8:00 - 10:00
St Pat's Ballroom

Use variation of parameters to find a particular solution $\vec{x}_p$ for the nonhomogeneous system

$\vec{x}' = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}\vec{x} + \begin{bmatrix} 1 \\ -5 \end{bmatrix}e^{2t}$, where $\Phi(t) = \begin{bmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{bmatrix}$ is a fundamental matrix of the associated homogeneous system.

$\det \Phi(t) = e^{-3t}(e^{2t}) - (-4e^{-3t})(e^{2t}) = 5e^{-t}$

$\Phi^{-1}(t) = \frac{1}{5e^{-t}}\begin{bmatrix} e^{2t} & -e^{2t} \\ 4e^{-3t} & e^{-3t} \end{bmatrix} = \frac{1}{5}\begin{bmatrix} e^{3t} & -e^{3t} \\ 4e^{-2t} & e^{-2t} \end{bmatrix}$

$\vec{x}_p = \Phi(t)\int \Phi^{-1}(t)\vec{F}\,dt$

$= \frac{1}{5}\Phi(t)\int\begin{bmatrix} e^{3t} & -e^{3t} \\ 4e^{-2t} & e^{-2t} \end{bmatrix}\begin{bmatrix} e^{2t} \\ -5e^{2t} \end{bmatrix}dt$

$= \frac{1}{5}\Phi(t)\int\begin{bmatrix} e^{5t} + 5e^{5t} \\ 4 - 5 \end{bmatrix}dt$

$= \frac{1}{5}\Phi(t)\int\begin{bmatrix} 6e^{5t} \\ -1 \end{bmatrix}dt$

$= \frac{1}{5}\Phi(t)\begin{bmatrix} \frac{6}{5}e^{5t} \\ -t \end{bmatrix}$

$= \frac{1}{25}\begin{bmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{bmatrix}\begin{bmatrix} 6e^{5t} \\ -5t \end{bmatrix}$

$= \frac{1}{25}\begin{bmatrix} 6e^{2t} - 5te^{2t} \\ -24e^{2t} - 5te^{2t} \end{bmatrix}$

$= \frac{6}{25}\begin{bmatrix} 1 \\ -4 \end{bmatrix}e^{2t} - \frac{1}{5}\begin{bmatrix} 1 \\ 1 \end{bmatrix}te^{2t}$