MTH 204
Quiz 9
10 Apr 2009

Name: Key

Section: C or F (circle one)

Read the directions carefully.
Write neatly in pencil and **show all your work**
(you will only get credit for what you put on paper).
You may use your homework solutions.
If you get stuck, free feel to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D

Exam 3: Friday, 24 Apr
7.4 - 8.3

By hand, calculate the eigenvalues and eigenvectors for the matrix $\begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix}$.

1. Find λ's

$0 = \det(A-\lambda I) = \lambda^2 - \text{Tr}A\,\lambda + \det A = \lambda^2 + 2\lambda - 80$

$= (\lambda+10)(\lambda-8)$

$\Rightarrow \lambda_1 = -10,\ \lambda_2 = 8$

2. Find $\vec{K}$'s

For $\lambda_1 = -10$, $(A-\lambda_1 I)\vec{K}_1 = \vec{0}$

$$\begin{bmatrix} 10-(-10) & -5 \\ 8 & -12-(-10) \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 20 & -5 & 0 \\ 8 & -2 & 0 \end{array}\right] \quad R_1 = \tfrac{5}{2}R_2$$

$\Rightarrow 4u_1 - u_2 = 0 \Rightarrow u_2 = 4u_1$ → FV = 1

$$\vec{K}_1 = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ 4u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \Rightarrow \text{eigenpair } \left\{-10, \begin{bmatrix} 1 \\ 4 \end{bmatrix}\right\}$$

For $\lambda_2 = 8$, $(A-\lambda_2 I)\vec{K}_2 = \vec{0}$

$$\begin{bmatrix} 10-8 & -5 \\ 8 & -12-8 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 2 & -5 & 0 \\ \cancel{8} & \cancel{-20} & \cancel{0} \end{array}\right] \quad R_2 = 4R_1$$

$\Rightarrow 2u_1 - 5u_2 = 0 \Rightarrow u_2 = \frac{2}{5}u_1$ → FV = 5

$$\vec{K}_2 = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ \frac{2}{5}u_1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \Rightarrow \text{eigenpair } \left\{8, \begin{bmatrix} 5 \\ 2 \end{bmatrix}\right\}$$

Bonus (2pts): Do $\vec{w} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}e^{-10t}$ and $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{-8t}$ form a FSS to $\vec{x}' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix}\vec{x}$? State why or why not.

No. $\vec{v}$ is not a solution.