This is a closed-book, closed-notes exam. The only items you are allowed to use are writing implements. Mark each sheet of paper you use with your name and clearly indicate the problem number.

The max number of points per question is indicated in square brackets after each question. The sum of the max points for all the questions is 55, but note that the max exam score will be capped at 50 (i.e., there are 5 bonus points, but you cant score more than 100%). Partial credit will be awarded, so show your work!

You have exactly 60 minutes to complete this exam. Keep your answers clear and concise while complete. Good luck!

1. You are building a highly reliable computer that is designed to still function even if some of its components fail. It has three CPUs, two network cards, and four hard drives, each of which can be 'functional' or 'failed'.
   (a) (3 points) How many possible states does this system have? List five of them.
   (b) (4 points) Suppose the system requires at least one functioning CPU, one functioning network card, and two functioning hard drives to work. How many of the possible system states result in a working system?
   (c) (4 points) Assume each system state is equally probable.[1] What is the probability that the system does not work?

2. (3 points) In a group of six people is it possible for each person to have exactly three friends in the group? Explain why or why not.

3. (4 points) You write up a survey for a psychology class testing, uh... the effect of liking ice cream flavors on survey responses. Suppose 20 people take your survey, and the survey software tells you the following information:

Response	*Count*
Chocolate	10
Vanilla	15
Mint	11
Chocolate and Vanilla	7
Chocolate and Mint	8
Vanilla and Mint	6

   (Each entry in the table counts the number of people who checked that box. Someone who checked 'chocolate' and 'vanilla' would be counted once for 'chocolate', once for 'vanilla', and once for 'chocolate and vanilla'.)

   How many people like all three kinds of ice cream?

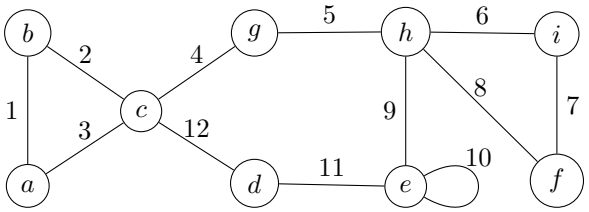
4. A local sandwich shop offers seven kinds of ingredients (turkey, ham, roast beef, lettuce, tomatoes, onions, and pickles) and three condiments (mayo, mustard, and hot sauce).
   (a) (4 points) How many different sandwiches can you order with three unique ingredients and two unique condiments?
   (b) (4 points) How does your answer to part 1 change if you can repeat ingredients but not condiments?[2]

5. (6 points) You and your friends head over to the local pizza joint for a meeting of the Discreet Discrete Math Club.[3] If someone wishes to join the club, they must come to a meeting and tell the members a true fact derived from some mathematical knowledge.

   While you are all engrossed in a hushed discussion of the finer points of binary trees, a stranger approaches the table and says, "At least three of you will order pizzas that share a topping." If there are seven of you at the table and the pizza joint offers three kinds of pizza topping, should you let the stranger join your club? Why or why not?

[1] Hopefully this isn't the case in reality!
[2] Triple ham, that's fine; triple mayo... ew.
[3] The Discrete Discreet Math Club meetings are rather boring as everyone has to sit at their own table.

6. Look at this graph:



   (a) (3 points) Write a path from b to h.

   (b) (8 points) Find an Euler circuit for this graph following the algorithm shown in class.

7. Let G be the graph given by the following adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

   (a) (4 points) Draw G.

   (b) (2 points) What is the total degree of G?

   (c) (6 points) Is G connected? Why or why not? Your answer should incorporate what it means for a graph to be connected.