
Zbl 1025.34001

Bohner, Martin (ed.); Peterson, Allan (ed.)

Advances in dynamic equations on time scales. (English)

Boston, MA: Birkhäuser. xi, 348 p. EUR 88.00/net; sFr. 144.00 (2003).

The book is a continuation of the editors's book [Dynamic equations on time scales. Basel: Birkhäuser (2001; Zbl 0978.39001)].

A time scale (resp. measure chain) is a closed subset of the reals; the term "dynamic equations" is a generalized notion that contains the notions of ordinary difference equations (the set of integers as time scale) and ordinary differential equations (the set of reals itself as time scale). The operator generalizing the forward difference operator and the differential operator, respectively, is called the "Delta derivative" operator. It is known [*S. Hilger*, Result Math. 18, No. 1/2, 18-56 (1990; Zbl 0722.39001)] that one can develop much of one-dimensional calculus and ordinary equations within this generalized context. In particular, the theory of time scales unifies and extends the classical theories of ordinary differential and difference equations.

The book contains contributions of several authors working in the area of time scales. The book is self-contained; nevertheless a familiarity with calculus on time scales and knowledge of the various corresponding time discrete or time continuous versions of the relevant topics are recommended. The authors suggest the book for a second course in dynamic equations at the graduate level.

Chapter 1 by *Martin Bohner*, *Gusein Guseinov* and *Allan Peterson* is an introduction to the time scales calculus. It is addressed to readers who are not familiar with time scales. Basic notions and features of this calculus are presented.

Chapter 2 is written by *Elvan Akin-Bohner* and *Martin Bohner*. They consider generalized versions of some classical scalar ODDs such as particular linear equations, Euler-Cauchy equations and the logistic differential equation. It turns out that the structural background of times scales calculus is useful to discover more "natural" generalizations or analogs of continuous equations.

Chapter 3 is authored by *Douglas Anderson*, *John Bullock*, *Lynn Erbe*, *Allan Peterson* and *HoaiNam Tran*. Motivated by an idea of *Atici* and *Guseinov*, they present a dualized (time-reversed) version of time scales calculus. The counterpart of a dynamic equation is called a Nabla dynamic equation.

The author of Chapter 4 is *Kirsten Messer*. The Delta and Nabla derivatives are adjoint to each other. This leads to the idea to consider 2nd-order Delta-Nabla-operator Sturm-Liouville equations, which are selfadjoint then. Much of the Sturm-Liouville theory, such as an oscillation theorem, separation theorem, comparison theorem, or Reid roundabout theorem, naturally carry over to the generalized setting.

In Chapter 5, *Martin Bohner* and *Gusein Guseinov* transfer various classical notions of an integral, such as the Cauchy, Darboux, Riemann, Lebesgue integrals, to the time scale case. Within this generalized setting, they examine relations between these integrals.

In Chapter 6, *Elvan Akin-Bohner*, *Ferhan Atici* and *Billur Kaymakcalan* study separated boundary value problems and periodic boundary value problems. They show the existence of solutions by monotone iterative techniques. Also the so-called quasilin-

earization method is applied.
Chapter 7 by *Douglas Anderson*, *Richard Avery*, *John Davis*, *Johnny Henderson* and *William Yin* is devoted to positive solutions of boundary value problems. They apply rich functional analytic theory, especially operator theory, a great variety of fixed-point theorems in partially ordered Banach spaces, and transversality methods, in order to investigate linear and nonlinear eigenvalue and boundary value problems for dynamic equations on a time scale.
The title of Chapter 8 authored by *Paul Eloe* is "Disconjugacy and Higher order Dynamic Equations". After some preparations concerning generalized zeros, Rolle's theorem, and Wronskian determinants, a central theorem characterizing disconjugacy by existence of various interpolating systems of functions is proven. A discussion of Trench factorization, principal solutions, positivity of Green's functions, monotone iteration for higher-order equations follows. The chapter closes with a list of open problems.
In Chapter 9, *Ravi Agarwal*, *Martin Bohner* and *Donal O'Regan* apply fixed-point theory in Fréchet spaces to coupled initial value-boundedness problems on time scales. An essential role in this chapter is played by certain generalizations of the Leray-Schauder fixed-point theory and the Gronwall lemma.
The heart of Chapter 10 on "Symplectic dynamic systems" by *Ondrej Dosly*, *Roman Hilscher* and *Stefan Hilger* is the so-called Reid roundabout theorem. It states the equivalence of completely different descriptions of disconjugacy, which are positivity of a certain quadratic functional, absence of nontrivial oscillatory solutions and existence of symmetric solutions to the corresponding Riccati dynamic equation. The theorem is interesting with respect to time scales since it shows and explains certain dicrepancies arising in the discrete and continuous special cases. Essential notions and tools surrounding this theory are focal points, principal solution, conjoined basis, generalized zeros, Moore-Penrose inverse, and Picone's identity. At the end the authors add some considerations on symplectic flows and the Prüfer transformation.

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Keywords : dynamic equations; time scales; time scales; dynamic equations; measure chains

Classification :

∗34-02 Research monographs (ordinary differential equations)
39-02 Research monographs (functional equations)
34B45 Boundary value problems on graphs and networks