1. Let

\[ w_1 = \begin{bmatrix} 2/3 \\ -1/3 \\ -2/3 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}. \]

Find the lengths of \( w_1 \) and \( w_2 \), their inner product, the distance between them, the angle between them, and the orthogonal complement of \( W = \mathcal{L}(w_1, w_2) \). Is \( v \in W^\perp \)? Is \( v \in W \)? Find the orthogonal projection of \( v \) onto \( W \), as well as the minimal distance from \( v \) to \( W \).

2. Suppose that a force \( y \) is applied to one end of a spring that has its other end fixed, thus stretching it to a length \( x \). In physics, Hooke’s law states that (within certain limits) there is a linear relation between \( x \) and \( y \). That is, there are constants \( \alpha \) and \( \beta \) with \( y = \alpha + \beta x \). The coefficient \( \beta \) is called the spring constant. Use the following data to (least-square) estimate the spring constant.

\[
\begin{array}{c|c|c|c|c}
\text{Length } x \text{ (in.)} & 3.5 & 4.0 & 4.5 & 5.0 \\
\hline
\text{Force } y \text{ (lb.)} & 1.0 & 2.2 & 2.8 & 4.3 \\
\end{array}
\]

3. Compute the determinant of

\[
\begin{bmatrix}
0 & -1 & 0 & 1 \\
-2 & 3 & 1 & 4 \\
1 & -2 & 2 & 3 \\
0 & 1 & 0 & -2
\end{bmatrix}
\]

by a cofactor expansion along the fourth row.

4. Find all eigenvalues and corresponding eigenvectors for

\[
\begin{bmatrix}
0 & -2 \\
-3 & 1
\end{bmatrix}.
\]