No notes are allowed. No calculators are allowed (except Problem 6). These are exactly the same problems as in Exams 1 (Problems 1–4) and 2 (Problems 5–8). Each problem is 25 points (if completely correct), 20 points (if more than half correct), 10 points (if less than half correct), or 0 points (if not correct). The final grades will be posted this evening. Good luck.

1. Rewrite

$$u + 2v + 3w = 7, \quad 2u + 5v + 6w = 1, \quad 3u + 6v + 7w = 1$$

as an equation $Ax = b$, find the LDU Decomposition of A, find c such that $Lc = b$, and find x such that $DUx = c$. Give the solution of the original problem and check your solution.

2. Given are the two matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}.$$

Find (if existent) A^T, B^T, AB, BA, AA^T, $I - A$, $2B$, A^{-1}, B^{-1}.

3. Is the set of vectors in $\mathbb{R}^3$ that have zero as the third component a subspace of $\mathbb{R}^3$? How about the set of vectors in $\mathbb{R}^3$ that have a nonnegative number as the third component? (Prove your claims.)

4. Find the four fundamental subspaces of $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 7 & 8 \\ 3 & 6 & 14 & 12 \end{bmatrix}$.

5. Let

$$w_1 = \begin{bmatrix} 2/3 \\ -1/3 \\ -2/3 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$$

Find the lengths of w_1 and w_2, their inner product, the distance between them, the angle between them, and the orthogonal complement of $W = \mathcal{L}(w_1, w_2)$. Is $v \in W^\perp$? Is $v \in W$? Find the orthogonal projection of v onto W, as well as the minimal distance from v to W.

6. Suppose that a force y is applied to one end of a spring that has its other end fixed, thus stretching it to a length x. Use the following data to (least-square) estimate the spring constant.

Length x (in.)	3.5	4.0	4.5	5.0
Force y (lb.)	1.0	2.2	2.8	4.3

7. Compute the determinant of $\begin{bmatrix} 0 & -1 & 0 & 1 \\ -2 & 3 & 1 & 4 \\ 1 & -2 & 2 & 3 \\ 0 & 1 & 0 & -2 \end{bmatrix}$ by a cofactor expansion along the fourth row.

8. Find all eigenvalues and corresponding eigenvectors for $\begin{bmatrix} 0 & -2 \\ -3 & 1 \end{bmatrix}$.