19. Let \( A_{11} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, A_{12} = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}, B_{11} = \begin{bmatrix} 3 & -7 & -7 & 2 \end{bmatrix}, \) and \( B_{21} = \begin{bmatrix} -2 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix}. \) Next, let \( A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 2 \end{bmatrix} \) and \( B = \begin{bmatrix} 3 & -7 & -7 & 2 \\ -2 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix}. \) Show that \( AB = A_{11}B_{11} + A_{12}B_{21}. \) Also, state a general theorem that can be used to solve such problems.

20. We would like to find a \( 3 \times 3 \)-matrix that has each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 as its entries such that each row and each column and each of the two diagonals sums up to 15. Don’t solve this problem, but just describe it as a system \( Ax = b \) where \( x \in \mathbb{R}^9 \) is the vector that has the entries of the required matrix as its entries.

21. Find the inverse of the \( 5 \times 5 \)-matrix from Example 1.7 (b) (you may use Maple if you wish). Also, find the \( u_k, 1 \leq k \leq 5 \) if \( f(x) = 1 \) and if \( f(x) = x. \) Compare them with the values \( u(x) \) of the real solution of \( u''(x) = f(x), u(0) = u(1) = 0. \)

22. Work on all of the Review Exercises of Chapter 1 on pages 60–62.