19. Let $A_{11} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $A_{12} = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$, $B_{11} = \begin{bmatrix} 3 & -7 & -7 & 2 \end{bmatrix}$, and $B_{21} = \begin{bmatrix} -2 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix}$. Next, let $A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -7 & -7 & 2 \\ -2 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix}$. Show that $AB = A_{11}B_{11} + A_{12}B_{21}$. Also, state a general theorem that can be used to solve such problems.
20. We would like to find a 3×3-matrix that has each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 as its entries such that each row and each column and each of the two diagonals sums up to 15. Don't solve this problem, but just describe it as a system $Ax = b$ where $x \in \mathbb{R}^9$ is the vector that has the entries of the required matrix as its entries.
21. Find the inverse of the 5×5-matrix from Example 1.7 (b) (you may use Maple if you wish). Also, find the u_k, $1 \le k \le 5$ if $f(x) = 1$ and if $f(x) = x$. Compare them with the values $u(x)$ of the real solution of $u''(x) = f(x)$, $u(0) = u(1) = 0$.
22. Work on all of the Review Exercises of Chapter 1 on pages 60–62.