27. For each of the following matrices A, find the four fundamental subspaces. Also, draw pictures featuring the null space and the row space.

(a) $A = \begin{bmatrix} 2 & 1 \end{bmatrix}$; (b) $A = \begin{bmatrix} -2 & -1 \end{bmatrix}$; (c) $A = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$;

(d) $A = \begin{bmatrix} 4 & 2 \\ 1 & -1 \end{bmatrix}$; (e) $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$; (f) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$;

(g) $A = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$; (h) $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 3 \end{bmatrix}$; (i) $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \\ 3 & 3 & 2 \end{bmatrix}$.

28. Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

(a) Find the smallest subspace U_1 of R^3 that contains v_1.

(b) Find the smallest subspace U_2 of R^3 that contains v_2.

(c) Find the *sum* U of these two subspaces U_1 and U_2, that is, the set of all possible combinations $x + y$, where $x \in U_1$ and $y \in U_2$.

(d) Finally find a subspace U_3 that satisfies $U + U_3 = \mathbb{R}^3$ and $U \cap U_3 = \{0\}$.

29. Find the sum of the null space and the row space for each of the matrices A from Problem 27.

30. Show in general that the sum of two subspaces of a vector space is again a subspace.