1. Rewrite
\[ u + 2v + 3w = 7, \quad 2u + 5v + 6w = 1, \quad 3u + 6v + 7w = 1 \]
as an equation \( Ax = b \), find the \( LDU \) Decomposition of \( A \), find \( c \) such that \( Lc = b \), and find \( x \) such that \( DUx = c \). Give the solution of the original problem and check your solution.

2. Given are the two matrices
\[ A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 7 \end{bmatrix}. \]
Find \( AA^T \), \( B^T A \), \( I - A \), \( 2B \), \( A^{-1} \), \( R(A) \), \( N(A) \), \( R(B^T) \), and \( N(B) \).

3. Is the set of vectors in \( \mathbb{R}^3 \) that have zero as the second component a subspace of \( \mathbb{R}^3 \)? How about the set of vectors in \( \mathbb{R}^3 \) that have a nonnegative number as the second component? (Prove your claims, of course).

4. Let \( B \), \( C \), and \( X \) be real \( n \times n \)-matrices that satisfy
\[ X^T X + B^T X + X^T B + C = 0 \]
Show that under these assumptions \( C \) must be necessarily symmetric.