38. Find all solutions to $x_2 + 2x_3 = b_1$, $4x_3 + 2x_2 = b_2$, $3x_2 + 6x_3 = b_3$, and $4x_2 + x_1 + 8x_3 = b_4$ if

 (a) $b_1 = 2$, $b_2 = 2$, $b_3 = 4$, and $b_4 = 5$;

 (b) $b_1 = 6$, $b_2 = 2$, $b_3 = 4$, and $b_4 = 5$.

39. Suppose two matrices A and B satisfy $AB = 0$. Show that the column space of B is contained in the nullspace of A.

40. Decide whether the following vectors are linearly independent or linearly dependent. For (a), (b), and (c), also draw a picture.

$$\text{(a)}\ \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix}; \qquad \text{(b)}\ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}; \qquad \text{(c)}\ \begin{bmatrix} 7 \\ 11 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\text{(d)}\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}; \quad \text{(e)}\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}; \quad \text{(f)}\ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 9 \\ 10 \\ 11 \\ 12 \end{bmatrix}.$$

41. For which choices of a, b, c, d, e, f are $\begin{bmatrix} a \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} b \\ c \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} d \\ e \\ f \\ 0 \end{bmatrix}$ linearly independent?

42. Given are three linearly independent vectors v_1, v_2, v_3. Are the following vectors lineary independent?

 (a) v_1, $v_2 + v_3$, $v_1 + v_2 + v_3$;

 (b) v_1, $v_1 + v_2$, $v_1 + v_2 + v_3$.

43. Find a basis and the dimension of each of the subspaces from Problem 26. Also, find matrices A and B such that each of these subspaces equals to the column space of A and to the nullspace of B.

44. Find the ranks of the following matrices. Also find a basis and the dimension of the four fundamental subspaces of each of the matrices.

 (a) All the matrices from Problem 30;

 (b) The 4×4-matrix on Page 104 of the textbook;

 (c) The 7×7-matrix on Page 477 of the textbook;

$$\text{(d)}\ A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}; \qquad \text{(e)}\ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{bmatrix}; \qquad \text{(f)}\ A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix};$$

$$\text{(g)}\ A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}; \quad \text{(h)}\ A = \begin{bmatrix} 1 & 2 & 0 & 3 & 5 \\ 0 & 0 & 1 & 4 & 6 \end{bmatrix}; \quad \text{(i)}\ A = \begin{bmatrix} 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$