13. Suppose p and g are continuous on some interval I. Let $t_0 \in I$ and y_0 be a real number. Show that the initial value problem $y' + p(t)y = g(t)$, $y(t_0) = y_0$ has exactly one solution, and give a formula for this solution.
14. A tank has ten gallons of water in which two pounds of salt has been dissolved. Brine with 1.5 pound of salt per gallon enters at three gallons per minute, and the well-stirred mixture is drained out at a rate of four gallons per minute. Find the amount of salt in the tank at any time.
15. Assume $y : (0, \infty) \to (0, \infty)$ is continuous with $\int_1^x \frac{y(t)}{t} \mathrm{d}t = y(x) - \frac{x^2+1}{2}$ for all $x > 0$. Find y.
16. At the beginning of the year, you have \$10,000 on your account. Assume you get 12% interest at the end of the year, how much money do you have at the end of the year? If you get twice 6% (after half a year, and then at the end of the year), how much money do you have finally? How about three times 4%? And four times 3%? And six times 2%? And twelve times 1%? And 360 times 1/30%? What do you observe about the results? Is there an upper bound if we make the time intervals even smaller? If so, what is the upper bound?
17. Solve the following initial value problems by separating the variables. Give the solutions explicitly and find their domains.
    (a) $y' = (1-2t)y^2$, $y(0) = -\frac{1}{6}$;
    (b) $y' = -\frac{x}{y}$, $y(1) = 1$;
    (c) $y' = \frac{x^2}{y}$, $y(0) = 1$;
    (d) $y' = \frac{x^2}{y}$, $y(0) = -1$;
    (e) $y' = \frac{3x^2-1}{3+2y}$, $y(0) = 1$;
    (f) $\sin(2t) + \cos(3y)y' = 0$, $y(\frac{\pi}{2}) = \frac{\pi}{3}$.
18. Let $N(t)$ be the number of individuals in a certain population at time t.
    (a) If we assume that N increases proportionally to the number of individuals currently present, give a differential equation for N and solve it. For an initial condition, assume that at time 0 the number of individuals is N_0.
    (b) Characterize the increasing, decreasing, and constant solutions from (a). Sketch the solutions. Find the limit of $N(t)$ as $t \to \infty$. Give an interpretation of all of your results.
    (c) Now we assume that N changes proportionally to the product of the number of individuals currently present and $(1 - \frac{N(t)}{K})$, where K is a constant. Give the corresponding ODE.
    (d) Solve the differential equation from (c) by separating the variables. For an initial condition, make the same assumption than in (a).
    (e) Characterize the increasing, decreasing, and constant solutions from (d). Sketch the solutions. Also, find the limit of $N(t)$ as $t \to \infty$. Give an interpretation of all of your results.
19. Give the solution y of the following problems explicitly:
    (a) $y' = y^2 + y + 1$;
    (b) $y' = y\sqrt{1-y}$, $y < 1$.
20. Find all solutions of $yy' = \sqrt{1 - (x^2 + y^2)} - x$, where $x^2 + y^2 < 1$, $y > 0$. Sketch the solutions. (Hint: Substitute $z(x) = \sqrt{x^2 + y^2(x)}$.)