38. Show that \( y_1(t) = t + 1 \) and \( y_2(t) = 2t + 4 \) solve the equation \( y = ty' + (y')^2 \) but that \( \alpha y_1 + \beta y_2 \) in general is not a solution. Why does this not contradict Theorem 3.5 as presented in the lecture?

39. Find two solutions of the equation \( t^2y'' - 2ty' + 2y = 0 \) such that their Wronskian is not zero (hint: try \( t^n \)). Calculate this Wronskian and give the interval where the solution is valid. Finally, find the solution of the equation that satisfies \( y(1) = 3 \) and \( y'(1) = 4 \).

40. Consider the problem \( t^2y'' + 3ty' + y = 0 \).
   
   (a) For which interval can we ensure the existence of a solution?
   
   (b) Find a solution \( y_1 \) of the form \( y_1(t) = t^\alpha \) for some real number \( \alpha \).
   
   (c) To find another solution, try \( y_2(t) = v(t)y_1(t) \) for some function \( v \).
   
   (d) Make sure that the Wronskian of \( y_1 \) and \( y_2 \) is not zero (if it is zero, try (a) and (b) again).

   Find this Wronskian.

   (e) Now find the solution that satisfies \( y(e) = \frac{e^2}{e} \) and \( y'(e) = \frac{e^2}{e^2} \).

41. Use steps similar as in the previous problem to solve \( 2t^2y'' + 3ty' - y = 0 \), \( y(1) = 3 \), \( y'(1) = 0 \).

42. Find the general solutions of the following equations:
   
   (a) \( y'' - 2y' + 2y = 0 \);
   
   (b) \( y'' + 6y' + 13y = 0 \);
   
   (c) \( y'' + 2y' + 2y = 0 \);
   
   (d) \( 4y'' + 9y = 0 \);
   
   (e) \( y'' + y' + y = 0 \);
   
   (f) \( y'' + 4y' + 6.25y = 0 \).

43. For each of the following initial value problems, find the solution.
   
   (a) \( y'' + 4y = 0 \), \( y(0) = 0 \), \( y'(0) = 1 \);
   
   (b) \( y'' + 4y' + 5y = 0 \), \( y(0) = 1 \), \( y'(0) = 0 \);
   
   (c) \( y'' - 2y' + 5y = 0 \), \( y(\frac{\pi}{2}) = 0 \), \( y'(\frac{\pi}{2}) = 2 \);
   
   (d) \( y'' - 2.5y' + y = 0 \), \( y(0) = 0 \), \( y'(0) = 1 \).

44. For the following equations, find one solution \( y_1 \) using the characteristic polynomial, and then try to find a second solution by trying \( y_2(t) = v(t)y_1(t) \) for some function \( v \) that needs to be determined. Make sure that the Wronskian of \( y_1 \) and \( y_2 \) is not zero. Then find the solution \( y \) with \( y(0) = 0 \) and \( y'(0) = 1 \).

   (a) \( y'' - 2y' + y = 0 \);
   
   (b) \( y'' - 4y' + 4y = 0 \);
   
   (c) \( y'' - 6y' + 9y = 0 \).