45. For the following equations, find one particular solution (hint: Try $a\mathrm{e}^{bt}$ or $a\sin(bt) + c\cos(dt)$).
    (a) $y'' - 2y' - 3y = 3\mathrm{e}^{2t}$;
    (b) $y'' + 2y' + 4y = 2\mathrm{e}^{-t}$;
    (c) $y'' + 2y' + 5y = 3\sin(2t)$;
    (d) $y'' + y = 3\sin(3t) + 4\cos(3t)$.
46. Find the solutions of the following initial value problems:
    (a) $y'' + y' - 2y = 2t$, $y(0) = 0$, $y'(0) = 1$;
    (b) $y'' + 4y = t^2 + 3\mathrm{e}^t$, $y(0) = 0$, $y'(0) = 2$;
    (c) $y'' + 4y = 3\sin(2t)$, $y(0) = 2$, $y'(0) = -1$;
    (d) $y'' + 2y' + 5y = 4\mathrm{e}^{-t}\cos(2t)$, $y(0) = 1$, $y'(0) = 0$.
47. Carefully use the variation of parameters technique in each of the problems below (exactly as presented in Ex. 3.14 (a) from the lecture) to find one particular solution of the following equations:
    (a) $y'' + y' - 2y = 2t$;
    (b) $y'' + 4y = 3\sin(2t)$;
    (c) $y'' + 2y' + y = 3\mathrm{e}^{-t}$;
    (d) $y'' + y = \tan(t)$;
    (e) $y'' + 4y' + 4y = t^{-2}\mathrm{e}^{-2t}$.
48. Work on Problems 28–32 in Section 3.7 of the textbook.
49. (First order difference equations)
    (a) Let $x_0 = 1$ and double this number to obtain x_1, double it again to obtain x_2 and so on. Find a formula for x_n, $n = 0, 1, 2, \ldots$. Use it to give x_{20}.
    (b) Let $x_0 = 1$ and muliply this number by p and add f to obtain x_1, multiply it again by p and add f to obtain x_2 and so on. Find a formula for x_n, $n = 0, 1, 2, \ldots$. Use it to give x_{20}.
50. (Second order difference equations)
    (a) Let $x_0 = x_1 = 1$. Add both numbers to obtain x_2, then add x_1 and x_2 to obtain x_3 and so on. Find a formula for x_n, $n = 0, 1, 2, \ldots$ (Hint: Try $x_n = r^n$ and use similar techniques as for differential equations). Use it to give x_{20}.
    (b) Let $x_0 = 0$, $x_1 = 1$. Multiply x_1 by $\frac{5}{2}$ and subtract x_0, to obtain x_2, then multiply x_2 by $\frac{5}{2}$ and subtract x_1 to obtain x_3 and so on. Find a formula for x_n, $n = 0, 1, 2, \ldots$. Use it to give x_{20}.