45. For the following equations, find one particular solution (hint: Try \(ae^{bt}\) or \(a\sin(bt) + c\cos(dt)\)).
   (a) \(y'' - 2y' - 3y = 3e^{2t}\);
   (b) \(y'' + 2y' + 4y = 2e^{-t}\);
   (c) \(y'' + 2y' + 5y = 3\sin(2t)\);
   (d) \(y'' + y = 3\sin(3t) + 4\cos(3t)\).

46. Find the solutions of the following initial value problems:
   (a) \(y'' + y' - 2y = 2t, \ y(0) = 0, \ y'(0) = 1\);
   (b) \(y'' + 4y = t^2 + 3e^t, \ y(0) = 0, \ y'(0) = 2\);
   (c) \(y'' + 4y = 3\sin(2t), \ y(0) = 2, \ y'(0) = -1\);
   (d) \(y'' + 2y' + 5y = 4e^{-t}\cos(2t), \ y(0) = 1, \ y'(0) = 0\).

47. Carefully use the variation of parameters technique in each of the problems below (exactly as presented in Ex. 3.14 (a) from the lecture) to find one particular solution of the following equations:
   (a) \(y'' + y' - 2y = 2t\);
   (b) \(y'' + 4y = 3\sin(2t)\);
   (c) \(y'' + 2y' + y = 3e^{-t}\);
   (d) \(y'' + y = \tan(t)\);
   (e) \(y'' + 4y' + 4y = t^{-2}\e^{-2t}\).


49. (First order difference equations)
   (a) Let \(x_0 = 1\) and double this number to obtain \(x_1\), double it again to obtain \(x_2\) and so on. Find a formula for \(x_n, n = 0, 1, 2, \ldots\). Use it to give \(x_{20}\).
   (b) Let \(x_0 = 1\) and multiply this number by \(p\) and add \(f\) to obtain \(x_1\), multiply it again by \(p\) and add \(f\) to obtain \(x_2\) and so on. Find a formula for \(x_n, n = 0, 1, 2, \ldots\). Use it to give \(x_{20}\).

50. (Second order difference equations)
   (a) Let \(x_0 = x_1 = 1\). Add both numbers to obtain \(x_2\), then add \(x_1\) and \(x_2\) to obtain \(x_3\) and so on. Find a formula for \(x_n, n = 0, 1, 2, \ldots\) (Hint: Try \(x_n = r^n\) and use similar techniques as for differential equations). Use it to give \(x_{20}\).
   (b) Let \(x_0 = 0, x_1 = 1\). Multiply \(x_1\) by \(\frac{5}{2}\) and subtract \(x_0\), to obtain \(x_2\), then multiply \(x_2\) by \(\frac{5}{2}\) and subtract \(x_1\) to obtain \(x_3\) and so on. Find a formula for \(x_n, n = 0, 1, 2, \ldots\). Use it to give \(x_{20}\).