33. Find the inverse Laplace transforms of

(a) $\dfrac{e^{-2s}}{s^2}$ (b) $\dfrac{1}{s^2} - e^{-s}\left(\dfrac{1}{s^2} + \dfrac{2}{s}\right) + e^{-4s}\left(\dfrac{4}{s^3} + \dfrac{1}{s}\right)$ (c) $\dfrac{2s}{s^2+4} - e^{-\frac{\pi s}{2}}\dfrac{3s+1}{s^2+9} + e^{-\pi s}\dfrac{s+1}{s^2+6s+10}$

34. Use the shifting theorem to find the Laplace transforms of

(a) $f(t) = u_1(t)(t^2+1)$ (b) $f(t) = \begin{cases} 2t+1 & 0 \le t < 2 \\ 3t & t \ge 2 \end{cases}$

(c) $f(t) = \begin{cases} 1 & 0 \le t < 2 \\ -2t+1 & 2 \le t < 3 \\ 3t & 3 \le t < 5 \\ t-1 & t \ge 5 \end{cases}$ (d) $f(t) = \begin{cases} \sin t & 0 \le t < \frac{\pi}{2} \\ \cos t - 3\sin t & \frac{\pi}{2} \le t < \pi \\ 3\cos t & t \ge \pi \end{cases}$

35. Use the Laplace transform to solve the following IVPs:

(a) $y'' + 2y' + 2y = 1$, $y(0) = -3$, $y'(0) = 1$;

(b) $y'' - 6y' + 9y = t^2 e^{3t}$, $y(0) = 2$, $y'(0) = 6$;

(c) $y'' + y = f(t)$, $y(0) = 2$, $y'(0) = -1$, where $f(t) = \begin{cases} 1 & 0 \le t < \frac{\pi}{2} \\ -1 & t \ge \frac{\pi}{2}; \end{cases}$

(d) $y'' - y = g(t)$, $y(0) = -1$, $y'(0) = 2$, where g is "ramp loading" between $(0,0)$ and $(1,1)$;

(e) $y'' + y = h(t)$, $y(0) = y'(0) = 0$, where $h(t) = \begin{cases} \cos(2t) & \frac{\pi}{4} \le t < \pi \\ 0 & \text{otherwise;} \end{cases}$

(f) $y'' - 2y' + y = \delta(t-1)$, $y(0) = 0$, $y'(0) = 0$;

(g) $y'' + 6y' + 5y = 3e^{-2t} + 2\delta(t-1)$, $y(0) = -3$, $y'(0) = 2$;

(h) $y'' + y = 1 + 2\delta(t-\pi) - 3\delta(t-2\pi)$, $y(0) = -1$, $y'(0) = 2$;

(i) $y'' - 2y' + y = k(t)$, $y(0) = a$, $y'(0) = b$;

(j) $y'' + 4y = k(t)$, $y(0) = a$, $y'(0) = b$;

(k) $y'' + 2y' + 2y = k(t)$, $y(0) = a$, $y'(0) = b$.

36. Show that $f * g = g * f$, $f * 0 = 0$, but in general we do not have $f * 1 = f$ or $f * f \ge 0$.

37. Exam #3 will be on Friday, November 7. To get ready for this exam, work through all of the problems 29–36 and in addition through the supplementary homework problems for Chapter 6: 1, 2, 3, 5–15, 17, 21, 26 of Section 6.1; 1, 3, 5, 7, 8, 11, 15, 19, 25 of Section 6.2; 1, 7, 13 of Section 6.3; 1, 5, 7 of Section 6.4; 1, 4, 9 of Section 6.5; 3, 4, 5, 9, 13, 16, 18 of Section 6.6.