Mathematics 204 Fall 2010 Exam II

1.[17] Find the general solution of $4y'' + 4y' + y = \dfrac{e^{-t/2}}{t^2}$ on the interval $t > 0$.

2.[17] Consider the differential equation $y''' - y = 7e^t$.

(a) Classify the differential equation by giving its order, stating whether it is linear or nonlinear, homogeneous or nonhomogeneous, and whether it has constant or variable coefficients.

(b) Which of the following solution methods are valid for solving this differential equation? If the method is valid, give a potential drawback in using this method.

Method of Undetermined Coefficients:_____

Variation of Parameters:_____

Laplace Transform:_____

(c) Find the general solution of the differential equation. The identity $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ may be useful.

3.[16] (Please use 32 ft/sec^2 as the acceleration of gravity in this problem.) A body weighing 8 pounds hangs from a vertical spring attached to the ceiling. At its equilibrium position, the body stretches the spring 1/2 ft from its natural length. The body is started in motion from the equilibrium position with an initial velocity of 4 ft/sec in the downward direction.

(a) Assuming that there is no damping and that the body is acted on by a downward external force of $3\cos(2t)$ pounds, set up, **BUT DO NOT SOLVE**, an initial value problem describing the motion of the body.

(b) If the given downward external force is replaced by a force of $3\cos(\omega t)$ pounds, find the value of the frequency ω which will cause resonance.

4.[16] Find the general solution of $t^2 y'' + 3ty' + y = 0$ on the interval $t > 0$.

5.[17] Use the definition of the Laplace transform, $\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$ for those values of s for which this improper integral converges, to find the Laplace transform of the function

$$f(t) = \begin{cases} t & \text{if} \quad 0 \le t < 1, \\ 1 & \text{if} \quad t \ge 1. \end{cases}$$

For which values of s is the Laplace transform of f defined?

6.[17] Use the Laplace transform to solve the initial value problem $y' + y = f(t),\ y(0) = 2,$ where

$$f(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < 1, \\ 1 & \text{if} \quad t \ge 1. \end{cases}$$

Caution: **NO CREDIT** will be awarded for any other method of solution.

Mathematics 204 Spring 2010 Exam II

1.[12] Solve the initial value problem: $y'' - 4y' + 3y = 0, \quad y(0) = -2,\ y'(0) = 5.$

2.[16] Find the general solution of the differential equation $y''' - y = 5e^x$. You may find useful the identity $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

3. (a) [5] Find the general solution of $x^2y'' - 2xy' + 2y = 0$ on the interval $0 < x < \infty$.

(b) [11] Find the general solution of the nonhomogeneous equation $x^2y'' - 2xy' + 2y = x^2$ on the interval $0 < x < \infty$, given that $y_1(x) = x$ and $y_2(x) = x^2$ form a fundamental set of solutions to the associated homogeneous equation on the interval $0 < x < \infty$.

4.[12] A certain spring hangs vertically from a rigid support. When a 3 pound mass is attached to the end of the spring, the mass stretches the spring 2 feet. Suppose the mass is pulled down 2 additional feet from the rest position and then released. Assuming air resistance (or the damping force) at any instant is equal to twice the instantaneous velocity of the mass, write **BUT DO NOT SOLVE**, an initial value problem describing the motion of the mass.

5.[14] Use the definition of the Laplace transform, $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$ for those values of s for which the improper integral converges, to compute the Laplace transform of the function given by

$$f(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < 1, \\ t-1 & \text{if} \quad 1 \le t < \infty. \end{cases}$$

(Caution: No credit will be awarded for any other method.) For which values of s is the Laplace transform of f defined?

6.[12] Find the inverse Laplace transform of $F(s) = \dfrac{s}{(s-2)(s-3)(s-6)}$.

7.[16] Use the Laplace transform to solve the initial value problem: $y'' + 4y = 8t, \quad y(0) = 0,\ y'(0) = 6.$ (Caution: No credit will be awarded for any other method.)

Mathematics 204 Spring 2010 Exam III

1.[20] Solve the initial value problem $y'' - 4y' + 5y = \delta(t - 2\pi), \quad y(0) = 0,\ y'(0) = 0.$ Please express your final answer without any unit step functions.