

1. Find the solution of the initial value problem  $y'' + y' + y = 0$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .
2. Use the variations of parameters technique exactly as presented in the lecture (showing all steps; not using a formula from the book; not using the method of undetermined coefficients etc.) to find the solution of  $y'' - 2y' + y = e^t$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .
3. A mass weighing 4 lb stretches a spring 6 inches. At time 0 the mass is released from a point 12 inches below the equilibrium position with upward velocity of 1 ft/sec.
  - (a) Determine the function  $x(t)$  which describes the subsequent free motion of the mass (ignoring any damping forces).
  - (b) Express  $x(t)$  in the form  $r \sin(\omega t + \theta)$ .
  - (c) Calculate the first three times when the mass is at the equilibrium position. If the mass is at the equilibrium position, how long does it take to be there next time? Will the spring ever be again at its starting position?
4. Find three linearly independent solutions of  $y''' - y'' - 2y' + 2y = 0$  and calculate their Wronskian at 0.
5. Find three linearly independent solutions of  $y_{n+3} - y_{n+2} - 2y_{n+1} + 2y_n = 0$  and calculate their Casoratian at 0 (hint: Just as in the second order case try  $y_n = r^n$  and find  $r$ ).