1. Determine the order of the following differential equations, and whether they are linear or nonlinear. Also verify directly that the given function is a solution of the equation:
   (a) $y' = t\sqrt{y}$ with $y(t) = \frac{t^4}{16}$;
   (b) $y'' + 16y = 0$ with $y(t) = 5\cos(4t) + 3\sin(4t)$;
   (c) $y' = 25 + y^2$ with $y(t) = 5\tan(5t)$;
   (d) $t^2y'' - ty' + 2y = 0$ with $y(t) = t\cos(\ln(t))$;
   (e) $y' = 2\sqrt{|y|}$ with $y(t) = t|t|$.
2. Use exactly the same steps as in Example 1.2 (a) from the lecture to find all solutions of the differential equation $y' + 4y + 2 = 0$. Also, give the solution y of this differential equation that satisfies $y(1) = 2$. Finally, let t_0 and y_0 be arbitrary real numbers and find the solution y of the differential equation that satisfies $y(t_0) = y_0$.
3. Find the ODE of all curves in the (x, y)-plane such that the area of the triangles between the tangent, the x-axis, and the y-axis is always constant. Solve this obtained equation (if you like) to find all those curves.
4. Find a differential equation of
   (a) first order that describes the family of straight lines passing through the origin;
   (b) second order that describes the family of circles passing through the origin.
5. Determine all values of r for which the given differential equation has solutions of the form $y(t) = e^{rt}$:
   (a) $y' + 2y = 0$;
   (b) $y'' + y' - 6y = 0$;
   (c) $y''' - 3y'' + 2y' = 0$.
6. Determine all values of r for which the given differential equation has solutions of the form $y(t) = t^r$, $t > 0$:
   (a) $t^2y'' + 4ty' + 2y = 0$;
   (b) $t^2y'' - 4ty' + 4y = 0$.
7. Draw a direction field for the given differential equation. Based on the direction field, determine the behavior of $y(t)$ as $t \to \infty$:
   (a) $y' = -1 - 2y$;
   (b) $y' = y(4 - y)$;
   (c) $y' = t + 2y$;
   (d) $y' = -\frac{2t+y}{2y}$.