18. Solve the following initial value problems by separating the variables. Give the solutions explicitly and find their domains.
    (a) $y' = (1-2t)y^2$, $y(0) = -\frac{1}{6}$;
    (b) $y' = -\frac{x}{y}$, $y(1) = 1$;
    (c) $y' = \frac{x^2}{y}$, $y(0) = 1$;
    (d) $y' = \frac{x^2}{y}$, $y(0) = -1$;
    (e) $y' = \frac{3x^2-1}{3+2y}$, $y(0) = 1$;
    (f) $\sin(2t) + \cos(3y)y' = 0$, $y(\frac{\pi}{2}) = \frac{\pi}{3}$.
19. Let $N(t)$ be the number of individuals in a certain population at time t.
    (a) If we assume that N increases proportionally to the number of individuals currently present, give a differential equation for N and solve it. For an initial condition, assume that at time 0 the number of individuals is N_0.
    (b) Characterize the increasing, decreasing, and constant solutions from (a). Sketch the solutions. Find the limit of $N(t)$ as $t \to \infty$. Give an interpretation of all of your results.
    (c) Now we assume that N changes proportionally to the product of the number of individuals currently present and $(1 - \frac{N(t)}{K})$, where K is a constant. Give the corresponding differential equation.
    (d) Solve the differential equation from (c) by separating the variables. For an initial condition, make the same assumption than in (a).
    (e) Characterize the increasing, decreasing, and constant solutions from (d). Sketch the solutions. Also, find the limit of $N(t)$ as $t \to \infty$. Give an interpretation of all of your results.
20. Consider the equation $y' = \frac{y^2+3ty}{t^2}$.
    (a) Find a function F such that $y' = F(\frac{y}{t})$.
    (b) Substitute $z = \frac{y}{t}$ and solve the resulting differential equation.
    (c) Find the solution of the original differential equation.
    (d) Based on the above, suggest a method on how to find the solution of an equation $y' = G(\frac{y}{t})$.
    (e) Test your method with the equation $y' = \frac{2y-t}{2t-y}$.
21. Give the solution y of the following problems explicitly:
    (a) $y' = y^2 + y + 1$;
    (b) $y' = y\sqrt{1-y}$, $y < 1$.
22. Find all solutions of $yy' = \sqrt{1-(x^2+y^2)} - x$, where $x^2+y^2 < 1$, $y > 0$. Sketch the solutions. (Hint: Substitute $z(x) = \sqrt{x^2+y^2(x)}$.)