

56. Calculate the Casoratian of any two solutions corresponding to any second order linear difference equations.
57. Develop a technique how to solve second order linear difference equations with constant coefficients in the case that the characteristic equation has two complex roots. Test this technique by finding solutions to the following initial value problems (compare to Problem 53):
- (a)  $y_{n+2} + 4y_n = 0$ ;
  - (b)  $y_{n+2} + 4y_{n+1} + 5y_n = 0$ ;
  - (c)  $y_{n+2} - 2y_{n+1} + 5y_n = 0$ .
58. Find the solutions of the following initial value problems:
- (a)  $y'' + y' - 2y = 2t$ ,  $y(0) = 0$ ,  $y'(0) = 1$ ;
  - (b)  $y'' + 4y = t^2 + 3e^t$ ,  $y(0) = 0$ ,  $y'(0) = 2$ ;
  - (c)  $y'' + 4y = 3 \sin(2t)$ ,  $y(0) = 2$ ,  $y'(0) = -1$ ;
  - (d)  $y'' + 2y' + 5y = 4e^{-t} \cos(2t)$ ,  $y(0) = 1$ ,  $y'(0) = 0$ .
59. Carefully use the variation of parameters technique in each of the problems below (exactly as presented in Ex. 3.19 (a) from the lecture) to find one particular solution of the following equations:
- (a)  $y'' + y' - 2y = 2t$ ;
  - (b)  $y'' + 4y = 3 \sin(2t)$ ;
  - (c)  $y'' + 2y' + y = 3e^{-t}$ ;
  - (d)  $y'' + y = \tan(t)$ ;
  - (e)  $y'' + 4y' + 4y = t^{-2}e^{-2t}$ .
60. Work on Problems 28–32 in Section 3.7 of the textbook.
61. Read Sections 3.8 and 3.9 of the textbook and work on at least five problems in each section.
62. Make sure that you can quickly solve any equation from the previous 61 problems, in particular equations of the following form:
- (a) First order linear differential equations with constant coefficients;
  - (b) First order linear differential equations with arbitrary coefficients;
  - (c) First order linear difference equations with constant coefficients;
  - (d) Second order linear differential equations with constant coefficients (no matter whether the characteristic equation has two real zeros, only one zero, or two complex zeros).
  - (e) Second order linear difference equations with constant coefficients (no matter whether the characteristic equation has two real zeros, only one zero, or two complex zeros).