

1. Let $r_n = 3 \cdot 2^n - 4 \cdot 5^n$ for $n \in \mathbb{N}_0$.

(a) Enter the values in the boxes: $r_0 =$, $r_1 =$, $r_2 =$.

(b) Show $\forall n \in \mathbb{N} \setminus \{1\} \quad r_n = 7r_{n-1} - 10r_{n-2}$.

2. Let $f_1 = 1$, $f_2 = 2$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$.

(a) Enter the values in the boxes: $f_3 =$, $f_4 =$, $f_6 =$.

(b) Show that $\sum_{k=1}^n f_k = f_{n+2} - 2$ and $\sum_{k=1}^n f_k^2 = f_n f_{n+1} - 1$ hold for all $n \in \mathbb{N}$.

3. Let $X = \{1, 2, \dots, 14\}$ and $R = \{(x, y) \mid x, y \in X \text{ and } 5 \mid (x - y)\}$.

(a) Find all elements of R .

(b) Is R reflexive, symmetric, antisymmetric, transitive, a partial order, or an equivalence relation (if so, find all equivalence classes)?

4. Let $X = \{x_1, x_2, x_3, x_4\}$, $Y = \{y_1, y_2, y_3, y_4\}$, $Z = \{z_1, z_2, z_3\}$, and define $f : X \rightarrow Y$, $g : Y \rightarrow Z$, $h : Z \rightarrow X$ by $f(x_1) = y_1$, $f(x_2) = y_3$, $f(x_3) = y_4$, $f(x_4) = y_2$, $g(y_1) = z_1$, $g(y_2) = z_1$, $g(y_3) = z_3$, $g(y_4) = z_2$, $h(z_1) = x_1$, $h(z_2) = x_2$, and $h(z_3) = x_4$.

(a) Find $h \circ g \circ f$.

(b) Write “y” for “yes” and “n” for “no” in the boxes: f is one-to-one ; f is onto ; g is one-to-one ; g is onto ; h is one-to-one ; h is onto .

5. This exercise refers to a club consisting of six distinct men and seven distinct women. In how many ways can we select a committee consisting of

(a) five persons? Enter the number value here .

(b) three men and four women? .

(c) four persons that has at most one man? .

(d) four persons that has at least one woman? .

(e) four persons that has persons of both sexes? .