Instructions: Do scratchwork on a separate sheet, which will not be collected. There is no partial credit for 1(a) and 2. Just fill in the boxes. You need to show all work for 1(b) and 3. For 4, you may only show the graph, the labels, and the solution (do all calculations on the scratchwork sheet). Each question is 40 points (so there are 10 bonus points available).

1. Let Ackermann's Function A be defined by $A(m,0) = A(m-1,1)\ \forall m \in \mathbb{N}$, $A(m,n) = A(m-1, A(m,n-1))\ \forall m,n \in \mathbb{N}$, and $A(0,n) = n+1\ \forall n \in \mathbb{N}$.
   (a) $A(1,1) =$ ____, $A(2,2) =$ ____, and $A(2,3) =$ ____.
   (b) Use induction to show $A(1,n) = n+2$ for all $n \in \mathbb{N}_0$.
2. The solutions to the following initial value problems
   (a) $a_n = 2na_{n-1}\ \forall n \in \mathbb{N}$, $a_0 = 1$;
   (b) $b_{n+1} = 7b_n - 10b_{n-1}\ \forall n \in \mathbb{N}$, $b_0 = 5$, $b_1 = 16$;
   (c) $L_{n+2} = L_{n+1} + L_n\ \forall n \in \mathbb{N}$, $L_1 = 1$, $L_2 = 3$
   are $a_n =$ ____, $b_n =$ ____, and $L_n =$ ____.
3. Consider a second order linear homogeneous recurrence relation $y_{n+2} = a_n y_{n+1} + b_n y_n$. Suppose x_n and z_n are two solutions of the equation. Show that $p_n = x_n + z_n$ and $q_n = \alpha x_n$ ($\alpha \in \mathbb{R}$) are also solutions of the equation.
4. Suppose it costs \$10,000 to purchase a new car. The annual operating cost for a car during it's first year is \$300, during the second year \$500, during the third year \$800, during the fourth year \$1200, during the fifth year \$1600, and during the sixth year \$2200. The resale value of a one year old car is \$7000, of a two year old car \$6000, for a three year old car \$4000, for a four year old car \$3000, for a five year old car \$2000, and for a six year old car \$1000. Assuming that one has a new car at present, determine a replacement policy that minimizes the net costs of owning and operating a car for the next six years. Draw a graph and use the Dijkstra Algorithm to work on this problem.