(44) Find the solutions of the following initial value problems:
   (a) \( a_n = 2na_{n-1} \forall n \in \mathbb{N}, \ a_0 = 1; \)
   (b) \( a_{n+1} = 7a_n - 10a_{n-1} \forall n \in \mathbb{N}, \ a_0 = 5, \ a_1 = 16; \)
   (c) \( L_{n+2} = L_{n+1} + L_n \forall n \in \mathbb{N}, \ L_1 = 1, \ L_2 = 3; \)
   (d) \( \sqrt{a_{n+1}} = \sqrt{a_n} + 2\sqrt{a_{n-1}} \forall n \in \mathbb{N}, \ a_0 = a_1 = 1; \)
   (e) \( a_{n+1} = \sqrt{\frac{a_{n-1}}{a_n}} \forall n \in \mathbb{N}, \ a_0 = 8, \ a_1 = \frac{1}{2\sqrt{2}}. \)

(45) Find all solutions of the following recurrence relations:
   (a) \( a_n = 5a_{n-1} \forall n \in \mathbb{N}; \)
   (b) \( a_{n+1} = 2a_n + 8a_{n-1} \forall n \in \mathbb{N}; \)
   (c) \( a_{n+1} = 6a_n - 9a_{n-1} \forall n \in \mathbb{N}; \)
   (d) \( u_{n+2} = 7u_{n+1} - 16u_n + 12u_{n-1} \forall n \in \mathbb{N}. \)

(46) Use the variations of parameter technique to find the solutions of the following initial value problems:
   (a) \( y_{n+1} - ny_n = (n + 1)!, \ y_1 = 5; \)
   (b) \( y_{n+2} - 5y_{n+1} + 6y_n = 2^n, \ y_1 = 4, \ y_2 = 9. \)

(47) Consider the second order linear homogeneous recurrence relation \( y_{n+2} = a_n y_{n+1} + b_n y_n. \)
    Suppose \( x_n \) and \( z_n \) are two solutions of the equation. Show
    (a) \( p_n = x_n + z_n \) is also a solution of the equation;
    (b) \( q_n = \alpha x_n \) is also a solution of the equation (for any \( \alpha \in \mathbb{R} \)).