(44) Find the solutions of the following initial value problems:

(a) $a_n = 2na_{n-1}\ \forall n \in \mathbb{N},\ a_0 = 1;$

(b) $a_{n+1} = 7a_n - 10a_{n-1}\ \forall n \in \mathbb{N},\ a_0 = 5,\ a_1 = 16;$

(c) $L_{n+2} = L_{n+1} + L_n\ \forall n \in \mathbb{N},\ L_1 = 1,\ L_2 = 3;$

(d) $\sqrt{a_{n+1}} = \sqrt{a_n} + 2\sqrt{a_{n-1}}\ \forall n \in \mathbb{N},\ a_0 = a_1 = 1;$

(e) $a_{n+1} = \sqrt{\frac{a_{n-1}}{a_n}}\ \forall n \in \mathbb{N},\ a_0 = 8,\ a_1 = \frac{1}{2\sqrt{2}}.$

(45) Find all solutions of the the following recurrence relations:

(a) $a_n = 5a_{n-1}\ \forall n \in \mathbb{N};$

(b) $a_{n+1} = 2a_n + 8a_{n-1}\ \forall n \in \mathbb{N};$

(c) $a_{n+1} = 6a_n - 9a_{n-1}\ \forall n \in \mathbb{N};$

(d) $u_{n+2} = 7u_{n+1} - 16u_n + 12u_{n-1}\ \forall n \in \mathbb{N}.$

(46) Use the variations of parameter technique to find the solutions of the following initial value problems:

(a) $y_{n+1} - ny_n = (n+1)!,\ y_1 = 5;$

(b) $y_{n+2} - 5y_{n+1} + ny_n = 2^n,\ y_1 = 4,\ y_2 = 9.$

(47) Consider a second order linear homogeneous recurrence relation $y_{n+2} = a_n y_{n+1} + b_n y_n$. Suppose x_n and z_n are two solutions of the equation. Show

(a) $p_n = x_n + z_n$ is also a solution of the equation;

(b) $q_n = \alpha x_n$ is also a solution of the equation (for any $\alpha \in \mathbb{R}$).