5. Are the following statements true or false? Prove your claim.
   (a) $\forall x \in \mathbb{R}\ x^2 - 9 = 0$;
   (b) $\exists x \in \mathbb{R}\ x^2 - 9 = 0$;
   (c) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ x = y^2$;
   (d) $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ xy = 0$;
   (e) $\forall \varepsilon > 0\ \exists N \in \mathbb{N}\ \frac{1}{N} < \varepsilon$;
   (f) $\forall \varepsilon > 0\ \exists \delta > 0\ \forall x \in (0, \delta)\ x^2 \in (0, \varepsilon)$;
   (g) $\forall n \in \mathbb{N}\ 3|n^2 \to 3|n$;
   (h) $\exists x \in \mathbb{Q}\ x^3 = 2$;
   (i) $\exists x \in \mathbb{Q}\ x^2 = 3$;
   (j) $\exists x \in \mathbb{Q}\ x^3 = 3$.
6. Work on problems 12–16 of Section 1.3 of the textbook.
7. Prove the rules of inference for propositions given in Figure 1.4.1 in the textbook.
8. Work on problems 21–25 of Section 1.4 of the textbook.
9. Prove the following statements using the Principle of Mathematical Induction:
   (a) $\forall n \in \mathbb{N}\ \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$;
   (b) $\forall n \in \mathbb{N}\ \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$;
   (c) $\forall n \in \mathbb{N}\ \sum_{k=1}^{n} (-1)^{k+1} k^2 = \frac{(-1)^{n+1} n(n+1)}{2}$;
   (d) $\forall n \in \mathbb{N} \setminus \{1, 2\}\ 2n + 1 \leq 2^n$;
   (e) $\forall n \in \mathbb{N} \setminus \{1, 2, 3\}\ 2^n \geq n^2$;
   (f) $\forall n \in \mathbb{N}\ 3^n \geq n2^n$;
   (g) $\forall n \in \mathbb{N}\ 5|(11^n - 6)$;
   (h) $\forall n \in \mathbb{N}\ 4|(6 \cdot 7^n - 2 \cdot 3^n)$;
   (i) $\forall x \geq -1\ \forall n \in \mathbb{N}\ (1 + x)^n \geq 1 + nx$.
10. Let $P(n) : \sum_{k=1}^{n} (2k) = (n + 2)(n - 1)$. Find the truth values of the following propositions:
    (a) $\forall k \in \mathbb{N}\ P(k) \to P(k + 1)$;
    (b) $\overline{\forall k \in \mathbb{N}\ P(k)}$.