9. Prove the following statements using the Principle of Mathematical Induction:
   (a) $\forall n \in \mathbb{N}\ \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$;
   (b) $\forall n \in \mathbb{N}\ \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$;
   (c) $\forall n \in \mathbb{N}\ \sum_{k=1}^{n} (-1)^{k+1} k^2 = \frac{(-1)^{n+1} n(n+1)}{2}$;
   (d) $\forall n \in \mathbb{N} \setminus \{1,2\}\ 2n+1 \le 2^n$;
   (e) $\forall n \in \mathbb{N} \setminus \{1,2,3\}\ 2^n \ge n^2$;
   (f) $\forall n \in \mathbb{N}\ 3^n \ge n2^n$;
   (g) $\forall n \in \mathbb{N}\ 5 | (11^n - 6)$;
   (h) $\forall n \in \mathbb{N}\ 4 | (6 \cdot 7^n - 2 \cdot 3^n)$;
   (i) $\forall x \ge -1\ \forall n \in \mathbb{N}\ (1+x)^n \ge 1 + nx$.
10. Let $P(n): \sum_{k=1}^{n} (2k) = (n+2)(n-1)$. Find the truth values of the following propositions:
    (a) $\forall k \in \mathbb{N}\ P(k) \to P(k+1)$;
    (b) $\overline{\forall k \in \mathbb{N}\ P(k)}$.
11. Work on problems 7–9 of Section 1.6 in the textbook.
12. We define the harmonic numbers as
$$H_n = \sum_{k=1}^{n} \frac{1}{k}.$$
    (a) Find H_1, H_2, H_3, H_4, H_5 as fractions $\frac{m}{n}$ with $m, n \in \mathbb{Z}$.
    (b) What is $H_{n+1} - H_n$ for $n \in \mathbb{N}$?
    (c) Prove by mathematical induction that $H_{2^n} \le 1 + n$ holds for all $n \in \mathbb{N}_0$.
    (d) Prove by mathematical induction that $\sum_{k=1}^{n} H_k = (n+1)H_n - n$ holds for all $n \in \mathbb{N}$.
13. By experimenting with some values of n, guess a formula for the sum
$$\sum_{k=1}^{n} \frac{1}{k(k+1)},$$
and then use mathematical induction to verify your formula.