40. Read Sections 5.1 and 5.2 of the textbook.

41. The population of Utopia increases 5 percent per year. At the beginning of 2000 the population was 10000. What was the population at the end of 1970?

42. Assume that a person invests \$3000 at 12% compounded annually. Let A_n be the amount of money at the end of n years.

    (a) Find A_1, A_2, A_3, and a recurrence relation that relates A_{n+1} to A_n for $n \in \mathbb{N}$.

    (b) Find A_n for all $n \in \mathbb{N}$.

    (c) How long will it take to double the initial investment?

43. Use "integrating factors" to find the solutions of the following initial value problems:

    (a) $y_{n+1} = 3y_n$, $y_0 = 4$;

    (b) $y_{n+1} = 3y_n - 2$, $y_0 = 1$;

    (c) $y_{n+1} = 4y_n - 5$, $y_4 = 3$;

    (d) $y_{n+1} = 1 - 2y_n$, $y_0 = 0$;

    (e) $a_{n+1} = 2(n+1)a_n$, $a_0 = 1$;

    (f) $a_{n+1} = 2(n+1)a_n + 1$, $a_0 = 1$.

44. Suppose we deposit \$1000 at the beginning of each year in an IRA that pays an annual rate of 7%. Let y_n be the money in the IRA at the end of the nth year. Find a recurrence relation for y and solve it.

45. Suppose we have \$10000 in a savings account that pays an annual interest rate of 3% and now start taking at the beginning of each year \$500. Let y_n be the money in the account at the end of the nth year. Find a recurrence relation for y and solve it. Will the account ever be empty? If so, when?

46. Find the solutions of the following initial value problems:

    (a) $y_{n+2} = y_{n+1} + 6y_n$, $y_0 = 1$, $y_1 = 2$;

    (b) $y_{n+2} = 7y_{n+1} - 12y_n$, $y_0 = 1$, $y_1 = 1$;

    (c) $L_{n+2} = L_{n+1} + L_n$, $L_0 = 1$, $L_1 = 3$;

    (d) $a_{n+2} = 6a_{n+1} - 9a_n$, $a_0 = -2$, $a_1 = 1$.

47. Find all solutions of the recurrence relation $u_{n+3} = 7u_{n+2} - 16u_{n+1} + 12u_n$.