19. Work on problems 67–82 of Section 2.2 in the textbook.

20. Show that for two sequences a_k and b_k we have

    (a) $\Delta(a_k + b_k) = \Delta a_k + \Delta b_k$;

    (b) $\Delta(a_k - b_k) = \Delta a_k - \Delta b_k$;

    (c) $\Delta(a_k b_k) = (\Delta a_k) b_k + (\Delta b_k) a_{k+1}$;

    (d) $\Delta\left(\frac{a_k}{b_k}\right) = \frac{(\Delta a_k) b_k - (\Delta b_k) a_k}{b_k b_{k+1}}$.

21. Let $f_1 = 1$, $f_2 = 2$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$.

    (a) Compute the first 10 elements of (the Fibonacci Sequence) f_n.

    (b) Show that $\sum_{k=1}^{n} f_k = f_{n+2} - 2$ and $\sum_{k=1}^{n} f_k^2 = f_n f_{n+1} - 1$ hold for all $n \in \mathbb{N}$.

    (c) Show that $f_{n+2}^2 - f_{n+1}^2 = f_n f_{n+3}$ holds for all $n \in \mathbb{N}$.

    (d) Show $f_n > \left(\frac{3}{2}\right)^n$ for all $n \in \mathbb{N} \setminus \{1, 2, 3, 4\}$ and $f_n < 2^n$ for all $n \in \mathbb{N}$.

    (e) Prove that $\sum_{k=1}^{n} f_{2k-1} = f_{2n} - 1$ and $\sum_{k=1}^{n} f_{2k} = f_{2n+1} - 1$ hold for all $n \in \mathbb{N}$.

22. Let $a_0 \in \left(0, \frac{1}{3}\right)$, and $a_{n+1} = a_n(2 - 3a_n)$ for $n \in \mathbb{N}$.

    (a) For your choice of a_0, compute a_k for $k \in \{1, 2, 3, 4, 5, 6\}$.

    (b) Show that $\{a_n\}$ is increasing.

    (c) Show that $\{a_n\}$ is bounded above.

    (d) Show that $\{a_n\}$ is convergent and compute its limit.

23. Let $a_0 \geq \sqrt{3}$, and $a_{n+1} = \frac{1}{2}\left(a_n + \frac{3}{a_n}\right)$ for $n \in \mathbb{N}$.

    (a) For your choice of a_0, compute a_k for $k \in \{1, 2, 3, 4, 5, 6\}$.

    (b) Show that $\{a_n\}$ is decreasing.

    (c) Show that $\{a_n\}$ is bounded below.

    (d) Show that $\{a_n\}$ is convergent and compute its limit.

24. Read Sections 2.4 and 2.5 of the textbook.