Final Exam, Math 2201, Dr. M. Bohner | Name: | Apr 29, 2002.

For the parts with boxes, please fill in the final results in these boxes. For those problems no partial credit is given; it's either true or false. If there is no box, please explain your solution in detail.

1. Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$.

   (a) $2A =$

   (b) $B + B^T =$

   (c) $AC =$

   (d) $ACB =$

   (e) $B^2 =$

2. Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$.

   (a) $\det A =$

   (b) $\operatorname{tr} A =$

   (c) $A^{-1} =$

   (d) The eigenvalues of A are [and]

   (e) The eigenvectors of A are $\begin{pmatrix} \ \end{pmatrix}$ and $\begin{pmatrix} \ \end{pmatrix}$

   (f) The eigenvalues of A^{-1} are [and]

   (g) The eigenvalues of $2A^2 + A - 3I$ are [and]

3. Suppose $r_0 = r_1 = 1$ and $r_{n+1} = r_n + 2r_{n-1}$ for each $n \in \mathbb{N}$. Use matrix powers to find a formula for r_n for each $n \in \mathbb{N}$.

4. Fill in the boxes:

IVP	integrating factor	solution $y(t)$
$y' = 5y - 1,\ y(0) = 2$		
$y' = -y + 4,\ y(1) = -1$		
$y' - y = 2te^{2t},\ y(0) = 1$		
$y' - 2y = e^{2t},\ y(0) = 2$		

5. Solve the problem $y' = \frac{t^2}{y}$, $y(0) = -1$ by separating the variables. Give the solution explicitly.

6. Let $N(t)$ be the number of individuals in a certain population at time t. If we assume that N increases proportionally to the number of individuals currently present, give a differential equation for N: $\boxed{\qquad\qquad\qquad}$. For an initial condition, assume that at time 0 the number of individuals is N_0. The solution is $\boxed{N(t) = \qquad\qquad}$. If $\boxed{N_0 \qquad\qquad}$, then the solution is increasing with $\boxed{\lim_{t\to\infty} N(t) = \qquad\qquad}$; If $\boxed{N_0 \qquad\qquad}$, then the solution is decreasing with $\boxed{\lim_{t\to\infty} N(t) = \qquad\qquad}$; If $\boxed{N_0 \qquad\qquad}$, then the solution is constant with $\boxed{\lim_{t\to\infty} N(t) = \qquad\qquad}$. Now we assume that N changes proportionally to the product of the number of individuals currently present and $(1 - \frac{N(t)}{K})$, where K is a constant. The corresponding ODE is $\boxed{\qquad\qquad\qquad}$. It can be solved using the $\boxed{\qquad\qquad\qquad\qquad\qquad \text{technique}}$. There are two constant solutions: $\boxed{N_1(t) \equiv \qquad}$ and $\boxed{N_2(t) \equiv \qquad}$.

7. Assume $y : (0, \infty) \to (0, \infty)$ is continuous with $\int_1^x \frac{y(t)}{t} \mathrm{d}t = y(x) - \frac{x^2+1}{2}$ for all $x > 0$. Find y.

8. Fill in the boxes (y_1 and y_2 should be linearly independent solutions):

equation	characteristic equation	zeros	$y_1(t)$	$y_2(t)$
$y'' - 3y' - 10y = 0$				
$y'' + 4y = 0$				
$y'' - 2y' + y = 0$				
$y'' - 2y' + 5y = 0$				
$y'' + 4y' + 5y = 0$				
$y'' - 2.5y' + y = 0$				

9. Use the variation of parameters technique to find one particular solution of $y'' + y' - 2y = 2t$.

10. Introduce $z = \binom{x}{y}$ and rewrite the equation as a system $z' = Az$. Find two linearly independent solutions z_1 and z_2:

equation	matrix A	eigenvalues of A	$z_1(t)$	$z_2(t)$
$x' = -4x + 2y,\ y' = -2.5x + 2y$				
$x' = 3x - y,\ y' = 9x - 3y$				
$x' = 5x + y,\ y' = -2x + 3y$				

11. Use Laplace transforms to solve the IVP $y' + 2y = t,\ y(0) = -1$.