6. Prove that $(A+B)^T = A^T + B^T$ and $(AB)^T = B^T A^T$ holds whenever A and B are matrices such that the addition/multiplication is defined.

7. For the following systems of equations, rewrite the systems as an equation $Ax = b$, do Gaussian Elimination and find the solution:

   (a) $2u + 4v = 3$, $3u + 7v = 2$;

   (b) $3u + 5v + 3w = 25$, $7u + 9v + 19w = 65$, $-4u + 5v + 11w = 5$;

   (c) $u + 2v + 3w = 39$, $u + 3v + 2w = 34$, $3u + 2v + w = 26$;

   (d) $u + 3v + 5w = 1$, $3u + 12v + 18w = 1$, $5u + 18v + 30w = 1$.

8. Work on Problems 33–38 of Section A.2 in the textbook.

9. Use the Gauss-Jordan Algorithm to find the inverses (if they exist) of the following matrices:

   (a) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $\begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix}$;

   (b) $\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$;

   (c) $\begin{pmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{pmatrix}$ and $\begin{pmatrix} 1 & 4 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{pmatrix}$.

10. Find the determinants of each of the following matrices:

    (a) The matrices from Problem 9;

    (b) $\begin{pmatrix} 2 & 5 & -3 & -2 \\ -2 & -3 & 2 & -5 \\ 1 & 3 & -2 & 2 \\ -1 & -6 & 4 & 3 \end{pmatrix}$ and $\begin{pmatrix} 6 & 2 & 1 & 0 & 5 \\ 2 & 1 & 1 & -2 & 1 \\ 1 & 1 & 2 & -2 & 3 \\ 3 & 0 & 2 & 3 & -1 \\ -1 & -1 & -3 & 4 & 2 \end{pmatrix}$.

11. Use determinants (i.e., Theorem 1.11 from the lecture notes) to find the inverses of the matrices from Problem 9.

12. Prove the properties of determinants (Theorem 1.12 from the lecture notes) for arbitrary 2×2-matrices.