39. Solve the following initial value problems:

(a) \( y'' - 3y' - 10y = 0 \). First, \( y(0) = 1 \), \( y'(0) = 0 \). Next, \( y(0) = 0 \), \( y'(0) = 1 \);

(b) \( 6y'' - 5y' + y = 0 \). First, \( y(0) = 4 \), \( y'(0) = 0 \). Next: \( y(0) = 0 \), \( y'(0) = 0 \);

(c) \( y'' + 3y' = 0 \), \( y(0) = -2 \), \( y'(0) = 3 \);

(d) \( 6y'' - 7y' + 2y = 0 \), \( y(0) = 0 \), \( y'(0) = 1 \);

(e) \( 2y'' - 3y' + y = 0 \), \( y(0) = 2 \), \( y'(0) = \frac{1}{2} \);

(f) \( y'' + 4y = 0 \), \( y(0) = 0 \), \( y'(0) = 1 \);

(g) \( y'' + 4y' + 5y = 0 \), \( y(0) = 1 \), \( y'(0) = 0 \);

(h) \( y'' - 2y' + 5y = 0 \), \( y\left(\frac{\pi}{2}\right) = 0 \), \( y'\left(\frac{\pi}{2}\right) = 2 \);

(i) \( y'' - 2.5y' + y = 0 \), \( y(0) = 0 \), \( y'(0) = 1 \);

(j) \( y'' - 2y' + y = 0 \), \( y(0) = 0 \), \( y'(0) = 1 \);

(k) \( y'' - 4y' + 4y = 0 \), \( y(0) = 0 \), \( y'(0) = 1 \);

(l) \( y'' - 6y' + 9y = 0 \), \( y(0) = 0 \), \( y'(0) = 1 \).

40. Consider the equation \( y'' = y \).

(a) Sketch the solutions \( c \) with \( y(0) = 1 \) and \( y'(0) = 0 \) and \( s \) with \( y(0) = 0 \) and \( y'(0) = 1 \).

(b) Show that \( c^2(t) - s^2(t) = 1 \) for all \( t \). Also, prove that \( c' = s \) and \( s' = c \).

(c) Draw the arch \( y(x) = -127.7c\left(\frac{x}{127.7}\right)^2 + 757.7 \). How high is it? How long is it’s base?

41. Find the Wronskian of the given pair of functions:

(a) \( e^{-2t} \) and \( te^{-2t} \); (b) \( e^{-2t} \) and \( \frac{3}{5}e^{-2t} \); (c) \( \cos t \) and \( \sin t \);

(d) \( \cosh t \) and \( \sinh t \); (e) \( t^n \) and \( t^m \); (f) \( t^n \) and \( mt^n \);

(g) \( t \) and \( te^t \); (h) \( \cos^2 t \) and \( 1 + \cos(2t) \).

42. If the Wronskian of \( y_1 \) and \( y_2 \) is \( 3e^{4t} \) and if \( y_1(t) = e^{2t} \), find \( y_2 \).

43. Consider the second order linear equation with constant coefficients \( ay'' + by' + cy = 0 \).

(a) Solve the IVP consisting of the equation and the initial conditions \( y(t_0) = y_0 \) and \( y'(t_0) = y'_0 \).

(b) Calculate the Wronskian of any two solutions of the equation.

Hint: You will need to work on three cases for each (a) and (b).