44. Find the general solution of \( y'' + 6y' + y' - 34y = 0 \) if it is known that \( y_1(t) = e^{-4t} \cos t \) is one solution.

45. Find the general solution of the equations in \# 39 (a)–(c) by using the method presented in Example 3.7 (f) from the lecture notes, i.e., rewrite the second-order equation as a system of first-order equations and find the eigenvalues and eigenvectors of the corresponding matrix.

46. Find two solutions of the equation \( t^2y'' - 2ty' + 2y = 0 \) such that their Wronskian is not zero (hint: try \( y(t) = t^\alpha \)). Calculate this Wronskian and give the interval where the solution is valid. Finally, find the solution of the equation that satisfies \( y(1) = 3 \) and \( y'(1) = 4 \).

47. Consider the problem \( t^2y'' + 3ty' + y = 0 \).

(a) Find a solution \( y_1 \) of the form \( y_1(t) = t^\alpha \) for some real number \( \alpha \).

(b) To find another solution, try \( y_2(t) = v(t)y_1(t) \) for some function \( v \).

(c) Make sure that the Wronskian of \( y_1 \) and \( y_2 \) is not zero (if it is zero, try (a) and (b) again). Find this Wronskian.

(d) Now find the solution that satisfies \( y(e) = \frac{e + 2}{6} \) and \( y'(e) = \frac{e - 2}{e^2} \).

48. Use steps similar as in the previous problem to solve \( 2t^2y'' + 3ty' - y = 0 \), \( y(1) = 3 \), \( y'(1) = 0 \).

49. (First order difference equations)

(a) Let \( x_0 = 1 \) and double this number to obtain \( x_1 \), double it again to obtain \( x_2 \) and so on. Find a formula for \( x_n, n = 0, 1, 2, \ldots \) Use it to give \( x_{20} \).

(b) Let \( x_0 = 1 \) and multiply this number by \( p \) and add \( f \) to obtain \( x_1 \), multiply it again by \( p \) and add \( f \) to obtain \( x_2 \) and so on. Find a formula for \( x_n, n = 0, 1, 2, \ldots \) Use it to give \( x_{20} \).

50. (Second order difference equations)

(a) Let \( x_0 = x_1 = 1 \). Add both numbers to obtain \( x_2 \), then add \( x_1 \) and \( x_2 \) to obtain \( x_3 \) and so on. Find a formula for \( x_n, n = 0, 1, 2, \ldots \) (Hint: Try \( x_n = r^n \) and use similar techniques as for differential equations). Use it to give \( x_{20} \).

(b) Let \( x_0 = 0, x_1 = 1 \). Multiply \( x_1 \) by \( \frac{5}{2} \) and subtract \( x_0 \), to obtain \( x_2 \), then multiply \( x_2 \) by \( \frac{5}{2} \) and subtract \( x_1 \) to obtain \( x_3 \) and so on. Find a formula for \( x_n, n = 0, 1, 2, \ldots \) Use it to give \( x_{20} \).