Exam #3, Math 2201, Dr. M. Bohner | Name: **Solution Key** | Apr 23, 2002.

Part A: Fill in **only** the boxes and do your work on a separate sheet.

1. Introduce $z = \binom{y}{y'}$ and rewrite the equation as a system $z' = Az$. Find two linearly independent solutions z_1 and z_2:

equation	matrix A	eigenvalues of A	$z_1(t)$	$z_2(t)$
$y'' - 3y' - 10y = 0$	$\begin{pmatrix} 0 & 1 \\ 10 & 3 \end{pmatrix}$	**5, -2**	$\binom{1}{5}e^{5t}$	$\binom{1}{-2}e^{-2t}$
$6y'' - 5y' + y = 0$	$\begin{pmatrix} 0 & 1 \\ -1/6 & 5/6 \end{pmatrix}$	$1/2, 1/3$	$\binom{2}{1}e^{t/2}$	$\binom{3}{1}e^{t/3}$
$y'' + 3y' = 0$	$\begin{pmatrix} 0 & 1 \\ 0 & -3 \end{pmatrix}$	**0, -3**	$\binom{1}{0}$	$\binom{1}{-3}e^{-3t}$

2. Fill in the boxes (again y_1 and y_2 should be linearly independent solutions):

equation	characteristic equation	zeros	$y_1(t)$	$y_2(t)$
$t^2y'' - 2ty' + 2y = 0$	$(r-2)(r-1) = 0$	**1, 2**	**t**	t^2
$t^2y'' + 3ty' + y = 0$	$(r+1)^2 = 0$	**-1**	$1/t$	$\ln(t)/t$
$2t^2y'' + 3ty' - y = 0$	$(2r-1)(r+1) = 0$	**0.5, -1**	$\sqrt{t}$	$1/t$

Part B: For the remaining problems, show your work clearly, explaining each step. Use only the space allocated for each problem (use separate sheets of paper for additional work).

3. Let $x_0 = x_1 = 1$. Add both numbers to obtain x_2, then add x_1 and x_2 to obtain x_3 and so on. Find a formula for x_n, $n = 0, 1, 2, \ldots$ (try $x_n = r^n$ and use similar techniques as for differential equations). Use it to give x_{20}.

This example was worked out in class. The solution is

$$x_n = \left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}.$$

4. A mass weighing 2 lb stretches a spring 6 inches. At time 0 the mass is released from a point 8 inches below the equilibrium position with upward velocity of $\frac{4}{3}$ ft/sec.

   (a) Determine the function $x(t)$ which describes the subsequent free motion of the mass (ignoring any damping forces).

   (b) Express $x(t)$ in the form $r\sin(\omega t + \theta)$. Sketch x.

   (c) Find the period and amplitude of the motion.

   This example is worked out on page 175 of the textbook.

5. Find the general solution of the system $x' = -6x + 5y$, $y' = -5x + 4y$.

The solution of this problem is posted on the website.