Exam #1, Math 315, Dr. Bohner.  

1. Define the total variation of a function \( f \) on an interval \([a, b]\).
2. What does the symbol \( \text{BV}[a, b] \) mean?
3. Suppose \( f \) is nondecreasing on \([a, b]\). Find \( V^b_a f \).
4. Every \( f \in \text{BV}[a, b] \) can be written as the difference of two nondecreasing functions. How can these two functions be chosen?
5. When is a function \( f: [a, b] \to \mathbb{R} \) called “Riemann-Stieltjes integrable”?
6. State the “Fundamental Inequality for Riemann-Stieltjes Integrals”.
7. If \( f \) is continuous and \( g \) is 0 up to \( t \) but makes a jump to \( p \) at \( t \) and stays at \( p \) afterwards, find the Riemann-Stieltjes integral of \( f \) with respect to \( g \).
8. State the “Integration by Parts Formula” for Riemann-Stieltjes integrals.
9. State the “Main Existence Theorem” for Riemann-Stieltjes integrals.
10. For which kind of integrands and integrators did we define lower and upper sums? How are they defined? What are the lower and upper Riemann-Stieltjes integrals? What is the “Main Existence Theorem” in this situation?
11. Define “pointwise” and “uniform” convergence of a sequence of functions \( \{f_n\} \) to a function \( f \) on a set \( E \).
12. Show that \( \{x^n\} \) converges uniformly on \([0, 1/2]\) but not on \([0, 1]\).
13. State the “Cauchy Criterion” for uniform convergence.
14. Describe the “Weierstraß M-Test”.
15. State the three main theorems on continuity, integrability, and differentiability of the limit function. Also state the corresponding corollaries for series.
16. What is a norm? What is a metric? What is a Banach space?
17. Which norm makes \( C[a, b] \) into a Banach space?
18. What is an equicontinuous family of functions?
19. State the “Arzelà-Ascoli Theorem”.
20. State the “Weierstraß Approximation Theorem”.