Problems #1, Math 315, Dr. Bohner. 1/16/1, due 1/23, 9:30 am.

1. Are the following functions in BV$[a,b]$? If so, find their total variations on $[a,b]$.

$$f(x) = \begin{cases} 0 & \text{if} \quad x \le 1 \\ 1 & \text{if} \quad x > 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 0 & \text{if} \quad x \ne 1 \\ 1 & \text{if} \quad x = 1. \end{cases}$$

2. Find $\bigvee_0^{3\pi} \sin$.

3. Show that $f \in \mathrm{BV}[0,1]$, where f is defined by

$$f(x) = \begin{cases} x^2 \sin(1/x) & \text{if} \quad x \ne 0 \\ 0 & \text{if} \quad x = 0. \end{cases}$$

4. Show that $f \in \mathrm{C}[0,1] \setminus \mathrm{BV}[0,1]$, where

$$f(x) = \begin{cases} x \sin \frac{\pi}{2x} & \text{if} \quad x \ne 0 \\ 0 & \text{if} \quad x = 0. \end{cases}$$

   Also show that g, defined by $g(x) = f(x^2)$, is differentiable but not of bounded variation on $[0,1]$.

5. Prove that Lipschitz functions are of bounded variation on $[a,b]$.

6. Prove that any step function is of bounded variation on $[a,b]$.

7. Prove that if $f, g \in \mathrm{BV}[a,b]$, then $f+g, f-g, fg \in \mathrm{BV}[a,b]$.

8. If $f : [a,b] \to \mathbb{R}$ is a monotonic function, find v_f.

9. Find v_f for f defined by $f(x) = \begin{cases} x+1 & \text{if} \quad -1 \le x < 0 \\ x & \text{if} \quad 0 \le x < 1 \\ 1-x & \text{if} \quad 1 \le x \le 2. \end{cases}$

10. Find $v_{\sin} : [0, 2\pi] \to \mathbb{R}$.

11. Show $\mathrm{BV}[a,b] \subset \mathrm{R}[a,b]$.