83. Let $f$ be summable. Show that the set $X(f \geq \alpha)$ has measure less than or equal to $(1/\alpha) \int_X fd\mu$. This is Chebyshev’s inequality.

84. Let $f$ be summable with $\int_X fd\mu < \infty$. Show that $f$ is finite almost everywhere on $X$.

85. Suppose that $f$ attains the values $c_1, c_2, \ldots$ (countably many) on exactly the (measurable) sets $A_1, A_2, \ldots$. Calculate $\int_X fd\mu$.

86. Let $f$ be the Dirichlet function. Find $\int_{\mathbb{R}} fd\mu$, where the measure space is $(\mathbb{R}, \mathcal{A}_L, \mu_L)$.

87. Let $f$ be a sequence with nonnegative numbers. Find $\int_{\mathbb{N}} fd\mu$, where the measure space is $(\mathbb{N}, \mathcal{P}(\mathbb{N}), \mu)$ with $\mu(A) = |A|$ for $A \subset \mathbb{N}$.

88. Let $f_n$ be functions defined on $X = [0,1]$ by $f_n(x) = n$ for $0 < x < 1/n$ and 0 otherwise. Does the L-integral of $f_n$ converge to the L-integral of the limit of $f_n$ as $n \to \infty$?

89. Assume $f$ is integrable (on a complete measure space). If $g \sim f$, then show that $g$ is integrable.

90. Assume $f$ and $g$ are integrable with $f \sim g$. Show that the integrals of $f$ and $g$ are the same.

91. Let $f, g, h$ be integrable and $g, h$ be nonnegative with $f = g - h$. Show that $\int_X fd\mu = \int_X gd\mu - \int_X hd\mu$. 