Problems #11, Math 315, Dr. Bohner. 4/26/1, due 5/10, 9:30 am.

83. Let f be summable. Show that the set $X(f \geq \alpha)$ has measure less than or equal to $(1/\alpha)\int_X f d\mu$. This is Chebyschev's inequality.

84. Let f be summable with $\int_X f d\mu < \infty$. Show that f is finite almost everywhere on X.

85. Suppose that f attains the values $c_1, c_2, \ldots$ (countably many) on exactly the (measurable) sets $A_1, A_2, \ldots$. Calculate $\int_X f d\mu$.

86. Let f be the Dirichlet function. Find $\int_{\mathbb{R}} f d\mu$, where the measure space is $(\mathbb{R}, \mathcal{A}_L, \mu_L)$.

87. Let f be a sequence with nonnegative numbers. Find $\int_{\mathbb{N}} f d\mu$, where the measure space is $(\mathbb{N}, \mathcal{P}(\mathbb{N}), \mu)$ with $\mu(A) = |A|$ for $A \subset \mathbb{N}$.

88. Let f_n be functions defined on $X = [0,1]$ by $f_n(x) = n$ for $0 < x < 1/n$ and 0 otherwise. Does the L-integral of f_n converge to the L-integral of the limit of f_n as $n \to \infty$?

89. Assume f is integrable (on a complete measure space). If $g \sim f$, then show that g is integrable.

90. Assume f and g are integrable with $f \sim g$. Show that the integrals of f and g are the same.

91. Let f, g, h be integrable and g, h be nonnegative with $f = g - h$. Show that $\int_X f d\mu = \int_X g d\mu - \int_X h d\mu$.