12. Show that the Riemann-Stieltjes integral is unique, if it exists.
13. Prove that \( f \, f \, d \, g \) is linear in \( f \) and \( g \).
14. Find \( f \, f \, d \, g \), where \( g \) is a constant function.
15. Find \( f \, f \, d \, g \), where \( g \) is a step function.
16. Define \( f \) and \( g \) by

\[
f(x) = \begin{cases} 
0 & \text{if } 0 \leq x \leq 1 \\
1 & \text{if } 1 < x \leq 2
\end{cases}
\]

and

\[
g(x) = \begin{cases} 
0 & \text{if } 0 \leq x < 1 \\
1 & \text{if } 1 \leq x \leq 2.
\end{cases}
\]

On which of the intervals \([0, 1], [1, 2], [0, 2]\) is \( f \in R(g) \)? Evaluate each of the three integrals, if they exist.

17. Calculate each of the following integrals:

\[
\int_0^1 x^2 \, d[x], \quad \int_0^\pi x \, \cos x \, dx, \quad \int_0^1 x^3 \, dx, \quad \int_{-1}^2 \sqrt{x + 2} \, d[x].
\]

18. For \( n \in \mathbb{N} \) and \( f \in C[0, n] \), find \( \int_0^n f \, d[x] \), and use your result to derive Euler's summation formula:

\[
\sum_{k=0}^n f(k) = \int_0^n f(x) \, dx + \frac{f(0) + f(n)}{2} + \int_0^n \left( x - [x] - \frac{1}{2} \right) f'(x) \, dx.
\]

19. Show that refining a partition decreases its upper sum.